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THE INFLUENCE OF CLASSIFICATION AND SERIATION ABILITY ON THE MATHEMATICAL ACHIEVEMENT OF FIRST GRADERS

BY

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled, "The Influence of Classification and Seriation Ability on the Mathematical Achievement of First Graders," submitted by Marilyn McCormack, in partial fulfilment of the requirements for the degree of Master of Education.

ABSTRACT

The purpose of this study was to investigate the relationship between the ability of first grade children to classify and seriate and their achievement in mathematics.

A sample of five grade one classes totaling one hundred thirteen children was randomly selected from schools in the Edmonton Public School System. The age, sex, parents' occupations, and Detroit Beginners First Grade Intelligence Test score for each child was obtained from cumulative record files kept by the schools.

Each child was exposed to two testing experiences. The first test was given during the final week of May, 1968 and consisted of individual tasks designed to determine the level of logical functioning of each child. This logic test was administered individually. The tasks dealt with sorting, class inclusion, multiple seriation, and ordering. A second test, the Seeing Through Arithmetic Test (S.T.A.T.), was administered by the classroom teachers during the first week of June 1968.

The data collected were analyzed using a multiple linear regression technique. A F ratio was computed and decisions to accept or reject the null hypotheses were made at the .05 level of significance.

The results of this analysis showed logic test scores to be significantly and positively correlated with intelligence and most scores on the S.T.A.T. Sex and intelligence correlated significantly and positively with S.T.A.T. scores.

Implications for education and suggestions for further research were given at the conclusion of the study.

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CHAPTER I

THE PROBLEM

I. INTRODUCTION TO THE PROBLEM

Piaget (1965) hypothesizes that the development of the notions of cardinality and ordinality are aided by the development of certain logical systems. The present study is an attempt to validate Piaget's hypothesis through the investigation of the ability of children to integrate logical processes and successfully deal with mathematical situations.

By the age of approximately seven years children attain a functional ability to classify objects and order them in a series. The cardinal concept derives from a disregard of irrelevant group properties such as size, color, shape, and texture of objects. It also requires a consideration of the general attribute of the group that is known as "number." The cardinal concept typifies classificatory logic. The concept of ordinal number is based upon the proper placement of numbers in a sequential continuum necessitating a comparison and ordering among entities. Therefore the logic of the concrete operational stage provides the foundation upon which later mathematical experience rests by making possible the conceptualization of cardinal and ordinal number. Theoretically then, mathematical achievement is related to logical functioning.

Feigenbaum (1963) proposes that a child's possession of certain logical operations is related to his general intelligence. General intelligence certainly would seem to have an influence on academic proficiency. Certain implications might be made which connect logic and achievement. Hood (1962) hinted at this connection when he investigated arithmetical progress and conceptual development. He found "no instance of success in arithmetic and failure in the Piaget tests... in the same child." (Hood, 1962, p. 284).

In this study only the conceptual processes of classification and seriation will be investigated. The results will be compared with mathematical achievement test scores and subjective ratings by teachers of their pupils' arithmetic ability.

II. THEORETICAL FRAMEWORK

Propositional logic cannot usually be handled by children from the ages of seven to eleven. The concrete structure which is present at this stage depends on the logic of classes and the logic of relations. The former type is dependent upon class inclusion operations. These operations denote an ability to manipulate part-whole relationships within a set of categories. The processes employed in this ability are those of addition and subtraction, or reversibility, and multiplication, which is used to discriminate between two or more independent variables (Piaget, 1958). Without the

use of categories one falls victim to reasoning by juxtaposition, indicating a thought process in which one partial link follows another.

The logic of relations utilizes serial ordering. For this type of reasoning, the ability is needed to generalize along a linear dimension or arrange objects, or properties, in a series. Sometimes it is also imperative that a correspondence can be found between two independent series; an "implication" ($\longleftrightarrow$) must be established. The comparative notions which are vital in these sequential ordering relations are those of "greater than" ($>$) and "less than" ($<$). There is frequently a definite hierarchy of development concerning these relations, wherein weights are ordered later than lengths and volumes later than weights (Piaget, 1958).

Piaget (1958) has distinguished three developmental stages which appear in both these logical processes. During the first stage, when considering static situations, the child explains them in terms of their configuration at the moment, instead of the changes leading from one situation to another. When the child does consider transformations, they are assimilated to his actions and as such are not reversible operations. At this level the child prefers to organize information merely on the basis of his perceptions. By Stage II every new situation is seen as a familiar situation which has been slightly altered. This recognition allows for the assimila-

tion of new knowledge and for the reversibility of thought. The appearence of class inclusion and seriation indicate the beginning of hypothetical deductions. The gradual growth toward reasoning in terms of the potential, away from the actual, culminates in the third developmental stage. At this time reality becomes subordinate to possiblity, and a propositional logic materializes in which verbal statements are substituted for objects.

III. PURPOSE OF STUDY

It was the purpose of this study to explore the possibility that mastery of classificatory and serial logic can help children to become better achievers in mathematics. More specifically, an attempt was made to ascertain whether each specific type of operation, that is classification and/or sequential ordering, was more conducive to success on particular types of items on a mathematical achievement test.

A secondary purpose of this study was to determine if in practice logical processes and general intelligence can be separated.

IV. SIGNIFICANCE OF STUDY

The teaching of number commonly begins in the first year of school. The child's first encounters with number are generally restricted to cardinal and ordinal number situations.

It is fundamental that the processes essential to cardinal and ordinal conceptualizations be internalized prior to formal number instruction to percipitate meaningful learning.

Since mathematical comprehension and achievement are most frequently assessed by written examinations, it is imperative that any examiner view the results in light of the level of logical operations at which the examinee is functioning.

V. HYPOTHESES

1. The ability to classify and seriate is not a significant predictor of mathematical achievement, when differences in ability are taken into account.

2. The ability to classify is not a significant predictor of mathematical achievement, when differences in ability are taken into account.

3. The ability to seriate is not a significant predictor of mathematical achievement, when differences in ability are taken into account.

4. The ability to classify and seriate is not a significant predictor of mathematical achievement test scores, taking into account differences in ability, when the knowledge of and ability to apply certain basic concepts of arithmetic is being examined.

5. The ability to classify is not a significant predictor of mathematical achievement test scores, taking into account differences in ability, when the knowledge of and ability to apply certain basic concepts of arithemtic is being examined.

6. The ability to seriate is not a significant predictor of mathematical achievement test scores, taking into account differences in ability, when the knowledge of and ability to apply certain basic concepts of arithmetic is being examined.

7. The ability to classify and seriate is not a significant predictor of mathematicl achievement test scores, taking into account differences in ability, when the knowledge of basic number facts is being examined.

8. The ability to classify is not a significant predictor of mathematical achievement test scores, taking into account differences in ability, when the knowledge of basic number facts in being examined.

9. The ability to seriate is not a significant predictor of mathematical achievement test scores, taking into account differences in ability, when the knowledge of basic number facts is being examined.

10. The ability to seriate is not a significant predictor of success on test items dealing with seriation.

11. The ability to classify is not a significant predictor of success on test items dealing with seriation.

12. The ability to seriate is not a significant predictor of success on test items dealing with classification.

13. The ability to classify is not a significant predictor of success on test items dealing with classification.

14. The ability to classify and seriate is not a significant predictor of success on test items dealing with seriation.

15. The ability to classify and seriate is not a significant predictor of success on test items dealing with classification.

16. Teachers' subjective rankings of their students' mathematical ability is not a significant predictor of the students' ability to classify and seriate, when differences in ability are taken into account.

17. Teachers' subjective rankings of their students' mathematical ability is not a significant predictor of the students' ability to classify, when differences in ability are taken into account.

18. Teachers' subjective rankings of their students' mathematical ability is not a significant predictor of the students' ability to seriate, when differences in ability are taken into account.

VI. LIMITATIONS OF STUDY

When interpreting the data of this study, the following limitations should be kept in mind.

1. No exhaustive attempt was made to control the variables connected with the teacher such as number of years taught, amount of teacher training acquired, method of teaching employed, or attitude toward mathematics instruction.

2. No attempt was made to equate subjects on the basis of their families' socio-economic status.

3. The procedure used to determine the functional levels of logical operations was adapted from those materials and procedures employed by Piaget (1964). No effort was made to measure the validity or reliability of the testing instrument.

4. The sample used was assumed to be representative of urban children between the ages of seventy-nine to eighty-eight months.

5. No exhaustive attempt was made to control variations

in vocabulary levels of the subjects.

VII. DEFINITIONS OF SPECIAL TERMS

Logical Operations. Logical operations are those which deal with relations between discontinuous elements.

Sub-logical Operations. Sub-logical operations are those relating to elements which form part of a spatial continuum.

Concept Formation. Concept formation, in its simplest sense, indicates the realization of what things belong together.

Class Inclusion Operations. Class inclusion operations imply the ability to manipulate part-whole relationships within a set of categories.

Concrete Operations. Concrete operations are actions which have been internalized and integrated with other actions to form a general reversible system.

VIII. THE EXPERIMENTAL SETTING

The experimental design is fully reported in Chapter III. A brief summary follows below.

The sample consisted of five first grade classes, a total of one hundred thirteen children from each of five randomly selected schools in the Edmonton Public School System. An instrument to determine the level of logical functioning was adapted from the work of Piaget (1964) and administered

to each child on an individual basis during the final week of May, 1968.

The instrument consisted of four individual tasks and took ten minutes per child to complete. The testing apparatus required no training session prior to its use. Three of the tasks demanded non-verbal responses and the fourth a "yes" or " no" answer to two questions. The tasks concerned themselves with a sorting activity, and an inclusion process (additive composition), an ordering activity, and a multiple seriation process (additive arrangement).

During the first week of June, a group administered mathematics achievement test was written by all children under the supervision of the classroom teachers. These tests were then corrected.

All data collected were analyzed in accordance with the multiple linear regression model. The statistical test used was that of the F-ratio.

IX. ORGANIZATION OF THE PAPER

A detailed analysis of the literature pertinent to logical and numerical conceptualizations is presented in Chapter II. Chapter III relates descriptions of the experimental design, testing procedures, and methods of statistical analysis. Immediately following in Chapter IV are the results of the data analysis. Chapter V concludes the paper with a

summary, followed by a discussion, implications, and suggestions for further research.

CHAPTER II

REVIEW OF RELATED LITERATURE

This study dealt with the logical processes of classification and seriation as predictors of mathematical achievement. A random sample of first grade pupils was selected. Each pupil was given an individual test of Piaget-like tasks which dealt with classification and seriation abilities. One week later the same students were given a mathematical achievement test which was administered as a group test. A brief discussion of the relationship between logical processes and number concepts may elucidate the possible dilemma of a small child whose logical processes have not yet developed to the concrete operational stage.

Direct references can be made to the _Seeing Through Arithmetic 1_ text (1965), which will support the contention that classification and seriation are the essence of beginning number work. The elementary behaviors of "bringing together" and "taking apart" are in reality the vital anticipatory and retrospective processes necessary to logical class construction. The first grade program is directly involved with these actions through the operations of addition and subtraction. Emphasis is laid on the recognition and symbolization of additive (combining) and subtractive (separating) actions. The relationship between these activities is illustrated in pictured situations which accompany work in problem solving.

The ability to abstract the criterion is a prerequisite of successful classification. The first grade geometry program concerns itself primarily with curves and the descriptive terms of "open," "closed," "exterior," and "interior." To master such concepts a child must be capable of extracting commonalities from each representation, for example, of an "open curve." The cardinal idea of number is dependent upon classification. Cardinality is presented in the Seeing Through Arithmetic 1 program by means of set theory. Instruction is aimed at developing children's recognition of the equivalence of sets by one-to-one correspondence. Ultimately, it is expected that the children will recognize the number of objects in a set by grouping and regrouping the elements.

Seriation is closely connected with the ordinal use of counting numbers and with the relationship of betweenness for the numbers one through ten. As linear measurement is dealt with in the first grade program, it is essential that the ability to perform simple sequential ordering be operational. Even more, the capacity to execute multiple seriations is demanded by the study of capacity measure and the use of ordered pairs of numbers to indicate positions of objects within a set; both these topics are included in the current grade one program.

The purpose of this chapter is to review the operational system of logic as explained by Piaget and to connect this logical system with the development of the concept of

number.

I. THEORIES OF LOGIC

The history of logic is one of numerous explanatory transformations. At first logic was used to provide a causal description of psychological circumstances. This practice is known as logicism. Although this approach has passed from the scene, still today any appeal to intuition, that being any kind of psychological factor, in the field of logic is termed psychologism. Over the decades the study of logic has demanded an increase in deductive rigor and the formalization of the character of logical systems. This routine has culminated in our theories of modern logic (logistics). Logistics encompasses four major theories.

Platonism refers to the establishment of a correspondence between logic and a system of universals existing independently of experience and which are non-psychological in orgin. The belief that logical entities owe their existence and laws to a system of conventions, generally accepted rule, is called conventionalism. Some theorists see logic as a well-formed language which distinguishes empirical truths, or non-tautological relationships, and tautologies, or syntactical relationships. Together with appropriate semantics this distinction may be used to express these empirical truths. Operationalism, most characteristic of Piaget's work, is a two-pronged theory. Operations are seen to play an

indispensible role in logic, yet on the other hand, operations are actual psychological activities. The term "logic" as used in the theory of operationalism denotes a system based on an abstract algebra and demanding symbolic manipulations.

II. PIAGETIAN LOGIC

According to Mays (Piaget, 1957) the structure of Piaget's logic is composed of elements which connect propositions. The elements are "not" (negation), "and" (conjunction), "or" (disjunction), and "if...then" (implication). Any two propositions in the system can be either true or false. Every relation thus formed has an inverse, which allows a return to the original state; a reciprocal, which is the identical proposition negated; a correlate, which arises out of substituting disjunction for conjunction and vice versa; and an identity operation, which when performed on any proposition leaves it unchanged. Propositional structures combine inversion and reciprocity in a single system; both are forms of reversibility. That is to say, propositional operations enable one to distinguish two independent variables, each whose modifications can be canceled (inversion) but which also can compensate each other without cancellation (reciprocity). (Piaget, 1958). Inversion concerns the negation (-A) or inclusion (:A) of a class. Reciprocity concerns relations and involves elimination of a difference.

Simple classification is another aspect of Piaget's

logical structure. Classifications occurring by successive inclusions in terms of dichotomous divisions result in the additive groupments of classes. Classifications occurring as a function of partial complementaries expressing qualitative correspondences result in the multiplicative groupments of classes. Simple classification is based on a system defined by "composition" (A+A'=B), "inversion"(A=B-A'), "identity" (A-A=O), "tautology"(A+B=B), and "associativity"(A+(A'+B')=(A+A')+B').

Within Piaget's algebra of logic are also elementary groupments, which although limited in scope, are transformations exhibiting closure, inverses, identity operator, and the associative property. These elementary groupments are based on simple sets or products sets. There also exist propositional structures based on the sets of all sub-sets, combinations of which yield the final component of Piagetian logic known as lattices. The limiting condition of this combinatorial structure is that all relations join and meet. They join to form smaller classes ((A,B), (A,C), (A,D)), and they meet to form the largest of the classes (Z). Diagrammatically a lattice is represented as follows.

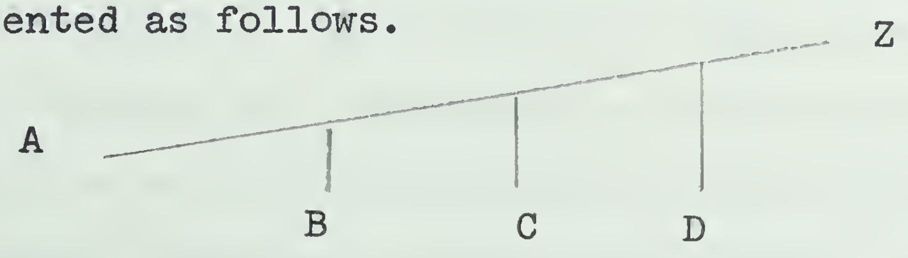

FIGURE 1

DIAGRAMMATIC REPRESENTATION OF A LATICE

Psychologically speaking operations are actions which are internalizable, reversible, and coordinated into systems characterized by laws which apply to the system as a whole. Accordingly, this structured whole, considered as the form of equilibrium of the subject's operational behavior, is of importance. The subject's capacity to perform operations and achieve equilibrium is accompanied by the construction of propositional functions.

III. STAGES OF DEVELOPMENT

The exact ontongeny of these logical operations is still not certain. Arthur (1944), as a result of the administration of what she terms "a non-verbal test of logical thinking," in which children had to reproduce symmetrical designs with the use of stencils, could only conclude that verbalizations accompanied the more advanced stages of operations. Braine (1959) could not establish any set order of logical reasoning either. All he was able to conclude was that logical reasoning consists of a group of "emergent" intellectual functions which all totaled become operational thinking. Braine discovered that where logical definitions are independent or derive from the same interpretation of a logical system, the corresponding intellectual processes will be found to develop in association with children's thinking; but precise ordering of these developments was impossible.

Piaget has achieved the most success in tracing operational development. He explains this development as consisting

of a set of logical definitions which are subsumed, with the intellectual processes, according to four stages. The first stage is characteristic of the first two years of life and is referred to as the sensori-motor stage. At this time the child performs ~~moror~~ motor actions. These actions have not yet been internalized to form representational thoughts. At this level of development object permanence is a function of an organized spatial field which is brought about by the coordination of the child's movements. Reversibility and associativity are reached through action. A child can return to the starting point and change the direction of his movements but without any accompanying symbolic representation.

The period of pre-operational thought is typically found in the child from two to seven years of age. Here appear the first symbolic functions of language, play, and imitation. As a consequence of this manifestation, representation formation becomes possible, enabling actions to become internalized and translated into thoughts. Yet these thoughts are not sufficient to abstract conservation principles at any higher level than previously. The effect of this absence of conservation is the nonexistence of operational reversibility.

At the concrete operational stage, approximately from seven to eleven years of age, the abilities possessed by the child are fragmentary- a child can classify, order serially, and form correspondences between objects but only at the expense of the whole. This prohibits complete generalization

but does allow for "mobile" equilibrium (reversibility) and multiplicative associations (classification according to two criteria). The operations carried out at this stage are still performed on the objects themselves. These formal operations are not yet completely dissociated from the now familiar actions of combining, dissociating, ordering, and matching. These finally give rise to logical operations which mark the fourth developmental stage.

Unlike reasoning at the concrete stage, which is one partial link after another, formal (propositional) reasoning proceeds by hypothesis. The thought of the child at this stage starts from theory so as to establish actual relationships between things. This "hypothetico-deductive reasoning" draws out the implications of possible statements and yields a synthesis of the possible and the necessary. The logic of this stage is concerned with propositions as well as with objects. This access to propostional operations brings on many new functions. For instance, one is now able to distinguish two independent variables, each of whose modifications can be cancelled (inversion) but which also can compensate each other without cancellation (reciprocity). This stage is marked by operational schemata consisting of combinatorial operations, proportions, equilibrium, probabilities, correlations, and the afore-mentioned multiplicative compensation.

IV. LOGICAL THOUGHT PROCESSES OF THE CONCRETE-OPERATIONAL STAGE

The present study concerns children who are possibly

functioning on the concrete operational level. Two forms of thinking central to this stage are classification and seriation; both these processes are dependent upon the formation of invariant concepts. The conceptualizing ability of children develops from a simple to a complex level as age increases (Vinacke, 1952). Feigenbaum (1963) admits to this also, but is not convinced that age accounts completely for differences in developmental levels. He is of the opinion that "... there is an interplay between general intelligence and possession or propensity for assimilations of the logical operations involved..." (Feigenbaum, 1963, p. 431). Vinacke (1952) clearly indicates this relationship by defining the ability to form and use concepts as one of the variables of intelligence. Thus part of the reason that mental age increases during the period of growth is that the ability to conceptualize increases. He attributes the effects of training and experience to be as influential in the development of concept formation as those of intelligence. Socio-economic status is interperted as having a low relation to conceptualization, whereas vocabulary is considered to be of primary importance. Kessen and Kuhlman (eds., 1962) concede that language plays a necessary, but not a sufficient, part in concept formation. They maintain that the component operations consituting logical classes show evidence of being linked by a markedly continuous progression through such elementary behaviors as "to bring together" and "to take apart." These anticipatory and retrospective processes precede and go beyond the use of linguistic association or connection. There is

something more to concept formation, that being the physical and mental mobility of operations.

Piaget (1964) agrees with Kessen and Kuhlman as to the significance of anticipation and retrospection; so much so that he has named them foresight and hindsight, respectively defining each as the process of internally carrying out action which will not be performed until later, thereby modifying present action and the process of revising earlier actions in light of those that have followed. Piaget considers flexibility in hindsight and foresight as an aid to the development of classification ability. The more mature a child becomes the more flexible is his handling of elements when classifying. Flexibility in hindsight is exhibited when the criterion can be altered either because some property has been noticed that was previously overlooked or because additional elements have been added to an existing classification. Flexibility in foresight is exhibited when a mental anticipation of a classification occurs or when the best possible classification is chosen over alternatives without overt trial-and-error. The early stage of graphic collections is a consequence of the lack of anticipation. The only anticipation witnessed is established in the course of collection making and once it is formed it becomes tied to the collection. When a child reaches the operational stage, he will not act until he has an anticipatory schema. This enables him to incorporate initial constructions into larger, more general ones.

Many studies conducted in the realms of logical thought strive to formulate a satisfactory means by which conceptualization can be tested. Hanfmann and Kasanin devised a procedure using wooden blocks which varied in color, shape, height, and size and which were to be grouped on a four-sectioned board. An analysis of the solutions attempted gave rise to four categories of responses. A "categorical" attitude is manifested by grouping all the objects which have one characteristic together and by altering the classification criteria when a contradiction is discovered. The "insight into the multiple possibilities of the choice" attitude is a realization of the possibilities so that an active, planned search for criterion can be carried out. The consideration of the "total system" by the subject prompts testing of every general characteristic to see if it will yield a four-way, mutually exclusive classification. Finally, classifications may be made on the bases of two characteristics, for example height and size, yielding four subclasses.

Reichard (1944) defined concept formation as a realization of what things belong together. On this basis a color-form test of conceptualization was developed in which twelve cardboard pieces of four different colors and three different shapes had to be sorted. Three methods of concept formation were demonstrated as attempted solutions. The first occurred at a concretistic level. Classifications were constructed on the basis of nonessential, incidental features. At the functional level use or value was used as a classificatory

criterion. When the general, abstract, conceptual level was attained classifications were made according to abstract properties or relations. These types of classifications will occur enumerable times in studies to be discussed further.

The concrete structure possessed by the typical seven year old depends on the logic of classes and relations. The operations of classification originate in essentially active behavior. This pre-classification, often refered to as "recognitive assimilation" (Piaget, 1964), is implicit in childish actions and judgments. Young children recognize that things belong to categories of wider generality and that things may simultaneously belong to two categories, but neither condition is sufficient to ensure correct inferences. Classificatory problems are connected with problems of inference, because classifications depend on the abstraction and retention of clear, unambiguous criteria. The classificatory behavior especially relevant for reasoning is the activity involved in abstracting criterion. This abstraction of criteria is a reversal of the actions of grouping, and the logical inferences generated derive exclusively from the abstraction of the subject's activity. Piaget himself says, "Logic... is not an innate characteristic of thinking, nor ... a mode of organization forced on us by the world as experienced. It is one that we construct by co-ordinating our own actions and abstacting the relations between them." (1964, p. xv)

The abstractions refered to are those of the extensive

and intensive properties of classes. Extension is based on a percise symbolism and quantification and defines the members of a class, such as the classes of foxes. Thus we are talking about the range of applicability of a concept. Intension is based on similarity relations and defines the properties common to the members of a class, such as the class fox. Thus we are talking about the meaning of a concept. The major problem in the development of classificatory behavior is the coordination of extension and intension. These difficulties with extension and intension come from irreversibility of thought. Once a child has conceived of a set of objects as being in one class (or possessing one property) he cannot return to the starting point of his mental construction and reconceive the same set of objects as being in a second class (or possessing a second property).

The lack of coordination between these two aspects of classification results in graphic collections being formed in response to the instruction to group objects that are alike. During this primitive stage classifications are framed in sensory-motor schemata concerned with noting and acting upon resemblences and differences. At first, there is no distinction between comprehension and extension. This comes with control of the logical quantifiers of "one", "some," and "all." For the present we shall concern ourselves with the refinement of "shifting" (altering criteria) which allows the subject to consider a collection from several vantage points.

This ability to "shift" eventually permits the allocation of objects to classes and the addition of new objects into existing structures. Bruner (1959) claims that in this indirect, limited sense the child is able to deal with the actual and the potential to the extent that provisions made are suitable for future use. Bruner (1962) distinguished three category types used in classification: conjunctive, defined by the joint presence of appropriate values of several attributes; disjunctive, defined by a choice of one or another attributes; relational, defined by a specific relationship between defining attributes. These three categorical types appear throughout the stages of classificatory development researched by Piaget (1964).

The typical child of age two and one-half to age five sorts objects globally. His attack is planless and proceeds in a stepwise manner. The child at this stage stands midway between the composite spatial object and the class itself. The Stage I child does not arrange elements in collections or sub-collections on the basis of similarity. He is unable to over-look the spatial configuration of the objects. He has poor command of intensive properties, being unable to isolate the properties belonging uniquely to members of one class which differentiate them from the members of other classes. There are numerous ways in which graphic collections may be assembled. The particular ones observed by Piaget (1964) follow. Most elementary is the formation of small partial alignments. These are linear arrangements in which some, but not all, of

the available material is used. Each object is related to the next one successively, with no anticipatory collection schema as a whole in mind. Very similar to these alignments are continuous ones marked by fluctuating criteria. This approach yields a linear pattern composed of sub-sets. As the child moves along the line he forgets what went on before so the criterion of similarity changes as successive comparisons are made. This problem reflects the child's difficulty in coordinating relations of similarity with part-whole relations. Intermediate between these alignments and collective and complex formations are situations in which children either neglect part-whole relations when they reinforce similarities or neglect similarities when they reinforce part-whole relations. Consequently, the whole tends to acquire properties of its own, which drive its classificatory function into the background. Collective groupings are two or three dimensional collections of similar elements which form a unified figure. Similar to these are complex groupings which are formed from heterogeneous elements and are based on geometrical form or situational content (descriptive meaning).

As a result of Piaget's investigations definite conclusions can be drawn. Classification entails coordination of part-whole relations with those of similarity and difference. These relations of similarity and difference exist at the level of graphic collections, but they are only applied to successive pairs of objects and as such are disconnected from the

part-whole relationships. These part-whole relationships are dependent on perceptual configuration, not the field of continuous sets. To state this in more classical terms, Stage I children cannot differentiate between properly logical and strictly sub-logical structures. Sub-logical operations bear on the parts of a spatial continuum while logical operations bear on the relations between separate elements. Proof of this incapacity to handle two concepts at once is the fact that children of this age can differentiate between similarity and belonging when not constructing collections of objects (Piaget, 1964).

Piaget (1964) attempts to explain the phenonmena of graphic collections as resulting from two difficulties. First, children confuse the relations of qualitative similarity with spatial contiguity, both in their alignments and in their complexobjects. The tendency is to introduce functional relations of belonging alongside those of similarity. Second, when little children set out to classify objects they end up with spatial and graphic wholes because there is some appreciation of intension (based on sensori-motor assimilation) but only one kind of extension (spatial or graphic extension of a perceptual whole). These two notions are incompatible.

The transition in classificatory development from Stage I to Stage II is governed by the appearence of hindsight and foresight. As soon as an assimilatory schema

becomes retroactive it takes on an anticipatory character; consistency with the past is impossible without making choices as to the future. Piaget (1964) found that the non-graphic collections of Stage II consisted of objects assigned to groups on the basis of similarity alone, but these collections were still juxtaposed and not used as a foundation for hierarchical class structuring. Often exhibited were small collections where a property was common to the group. Later these collections were united to form larger arrangements with more general properties in common. Such action is refered to as the "ascending method." In contrast to this procedure large collections with general properties sometimes were formed and these subsequently subdivided into smaller collections ("descending method"). At this stage nonfigural collections are characterized by similarity and subgrouping. A collection exists by virtue of the union of its elements in space and ceases to exist when its subcollections are dissociated. Piaget (1964) continued his experiment and formulated four types of non-graphic collections. The first type consists of a number of small collections based on different criteria, together with an unclassified heterogeneous remainder. Secondly, he found small collections based on a multiplicity of criteria, with no remainder or overlap. The third attack is typified by an elimination of fluctuations of criteria. This finally allows the fourth method of grouping which produces internal differentiations of a lower rank.

The hallmark of complete classificatory structure is

operational mobility allowing for a change of criterion, or a "shifting." When a child anticipates transformations (ascending and descending methods together) instead of concentrating on static results there is equilibrium, and he is said to have reached Stage III. In the classificatory behavior of a typical Stage III child are found all the characteristics of true classification:

1. no isolated elements
2. no isolated classes
3. class A includes all individuals having the property a
4. class A includes only individuals having the property a
5. classes are disjoint
6. a complementary class has its own characteristics
7. class inclusion in a hierarchy
8. extensional simplicity (inclusions reduced to a minimum)
9. intensional simplicity (similar objects form separate classes)
10. symmetrical subdivisions

Much research has been done concerning children's classificatory behavior using objects other than flat, geometrical shapes. Marian Annett (1959) investigated the sorting practices and explanations given by children when grouping pictures of common objects. She concluded that the method of classification used is determined by the subject's familiarity with the items, the range of the items, and the subject's purpose in making the classification. It is her assertion that "children first need to analyse the character-space before

they can attend to significant similarities" (1959, p.235). She noted five ways of classifying used by the children in her study. Sometimes no attempt was made to explain the classification, or else a simple statement that the objects belonged together was made. Often enumeration was relied upon. An analysis of the significant features was made from the total complex of attributes, but the child failed to consider the attributes of two objects simultaneously. This frequently occurs up to age seven. The objects are related to one another according to differences in juxtaposed form. Using physical contiguity in space as a classificatory criterion exists until age eight. This practice precedes similarity grouping, and objects are related to one another in terms of direct, concrete interactions involving place, time, animal activity, human activity, and serial contiguities. This practice is basically concerned with how the world is organized and what-goes-with-what in everyday experience. Similarity sorting elicits the specification of characteristics common to objects, such as place, activity, and structure. The final classificatory procedure identified was the assignment of class names. This method most frequently occurred with animal sortings, while vehicle and furniture sortings depended more heavily on similarity and contiguity respectively.

Jane Thompson (1941) sought to distinguish prefered methods of sorting for different grade levels. On the basis of her investigations using the Weigel-Color-Form Test and

the B.R.L. Sorting Test, it was concluded that regardless of grade level children generalize through a process of analysis and synthesis. Kagen and Lemkin (1961) have conducted a more recent study of children's conceptual behavior when involved with geometric shapes. Their testing materials were comprised of geometrical forms pasted on white paper; three different forms, four different colors, two different sizes. They found that boys and girls prefered form, color, and size as a basis for similarity grouping in that order. These results are somewhat contradictory to those of Brian and Goodenough. Brian and Goodenough (1929) found that children under three years of age group by form, but that between the ages of three to six they prefer color. Suchman and Trambasso (1966) also found this preference in three to six year olds. It is also Brian and Goodenough's contention that girls more frequently group by form, rather than color, than do boys.

Olver and Hornsby, in an article entitled "On Equivalence" (Bruner, 1966), review all aspects of classificatory behavior. They cite three bases for establishing equivalence-enactive representation where objects are seen as alike on the basis of a common role in some action, ikonic representation where items are grouped according to a perceptual kinship or likeness, and symbolic representation where equivalence is governed by the grammatical principles of synonyms, superordination, and syntactic substitutability. They suggest

there is a language frame for characterizing the basis of equivalence. This frame is perceptual in nature because equivalence can be based on immediate phenomenal qualities such as color, size, and shape or on the basis of position in time or space. The frame is functional in nature because of the consideration given to the use of items, what they do or what can be done to them. The affective nature of the frame is exhibited when objects are sorted in accordance with the emotions they arouse or a person's evaluation of them. The proposed frame possesses the nominal aspect of grouping in reference to a name that exists ready-made in the language. Very often this language frame takes on the disguise of fiat equivalence; one just says the items are alike or the same without giving a reason.

Olver and Hornsby make detailed remarks about general grouping structures. They identify superordinate groupings as those constructed on the basis of a common feature characteristic of items in the class. General superordinates reflect a common characteristic of items in the group. Itemized superordinates tell the reason each item qualifies. Therefore, it may generally be stated that functional attributes yield superordinates. Eight year old children prefer functional groupings and are egocentric in their notions of how an object is used. Complex structures make use of the attributes of an array and form local, rather than universal, rules for grouping. When related properties are found in objects, but the objects are not tied together in terms of

attributes shared, collections are formed. Edge matchings are a consequence of the formation of associative links between neighboring items. Key-ring groupings denote taking an item and linking all others to it by choosing attributes that form relations between the central item and each of the others. Associations are the linkage of two items and the use of a bond between these items as a nucleus for the addition of other objects. The final type of complexive organization is that of multiple groupings. It appears that younger children more frequently classify items using complexes, which is not strange since complexes are founded on perceptible attributes. Thematic groupings are verbal in origin. Groups are formed on the criteria of how they fit in a sentence, story, or theme. Although Olver and Hornsby have implemented a detailed analysis of classificatory behavior, they have neglected to mention the problem of class inclusion that they must have encountered.

Because extension and intension are at first ill-defined and undifferentiated, young children cannot distinguish between the words "some" and "all." These words are partly extensive and partly intensive in their nature. When used by young children these words describe an element in terms of the whole object without any reference to the individual parts. Successful class inclusion operations are not possible unless the child uses categories. He must possess the ability to manipulate part-whole relationships within these categories through the use of addition and subtraction, known as reversibility,

and multiplication, used to discriminate between two or more independent variables. To state this more clearly, the inclusion of classes depends on an anticipatory schema essential for reversibility and an understanding of the quantitative relations of less than and greater than (Piaget, 1964). This is deceiving, due to the fact that the extension of a collection is not purely a quantitative notion. The notion of extension cannot be constructed without first qualifying the elements in the grouping. And it cannot be decided what are "all" and "some" without refering to the more or less common properties of the grouping - its intension. It is because extension presupposes intension and vice versa that confusion persists in some cases and not in others.

Logically, inclusion figures as a relation between two cases of class-membership. "All," the universal quantifier, when preceding a propositional form, signifies that all values for this form are true propositions; that is, the relation expressed in the form holds among any terms. "Some," the particular quantifier, indicates there exists at least one value of a variable in the form which will yield a true proposition (Langer, 1953). In terms of questions asked of children this means that whenever they are asked "Are all the B's As (where B=A+A')?" they invert the question and interpret it as "Are all the A's Bs?" They further alter this with a false quantification and finally answer the question "Are all the As all the Bs?"

According to Piaget (1964), at Stage I in their classificatory development children see no difference between "all" and "some." By Stage II an awareness exists that "some" has a different meaning from "all," but there is no fixation on one stable meaning for the word. The meaning of "some" becomes bound up with the number of elements instead of pointing to a relation between the part and the whole. In terms of the graphic and non-graphic phases discussed before, at the level of graphic collections children try to make up some kind of collective object in order to envisage "all the Xs." This is an oversimplification of "all." At this age it is easier to intuit about the majority of the elements and "all" is used to refer to the collection as a whole, seldom to the entirity of sub-sets. All children at the non-graphic level can do to decide whether all the Xs are Y is to ascertain whether or not the collection of Xs coincides with that of Ys (like the previous, "are all the As all the Bs?"). This tendency to substitute identity for inclusion produces sometimes correct and sometimes incorrect answers. To achieve genuine class-inclusion the child must exhibit conservation of the whole and a quantitative aspect must be maintained. This quantitative aspect of the logical concept of a class inclusion depends upon the prior formation of a hierarchical system of classes.

Kofsky (1966) has devised a scale according to which such a hierarchical system develops. The beginnings lie in resemblence sortings (look alikes), consistent sortings

(extends scope from two to more than two objects), and exhaustive sortings (extend to all objects that could be considered equivalent in some respect). The next phase entails conservation because the resulting groupings are transitory. The final stages procede from multiple class membership where objects do not exclusively belong in different categories, to horizontal classification where different groupings of objects are tried out, and finally ends in hierarchical classification. At this point a single attribute, or a combination of attributes, is choosen to construct successive classes. This progression may be outlined as seen in FIGURE 2. (Kofsky, 1966, p. 194)

Elkind (1961), as a result of his investigation into the additive composition of classes, concludes that the ability to include classes indicates the attainment of logical (abstract) class conceptions. He attributes children's difficulty in composing classes at Stage I and II to (1) their inability to think of the same set of elements in two classes at once (extension); the realistic character of children's thoughts cause them to take their mental construction of their classes for a real spatial entity; and (2) their inability to think of a single element as possessing two properties at once (intension); their conceptions of classes are not clearly differentiated from their object conceptions. Logical difficulties arise from an inability to group operations so that the whole remains invariant whatever the relationship between the whole and the part.

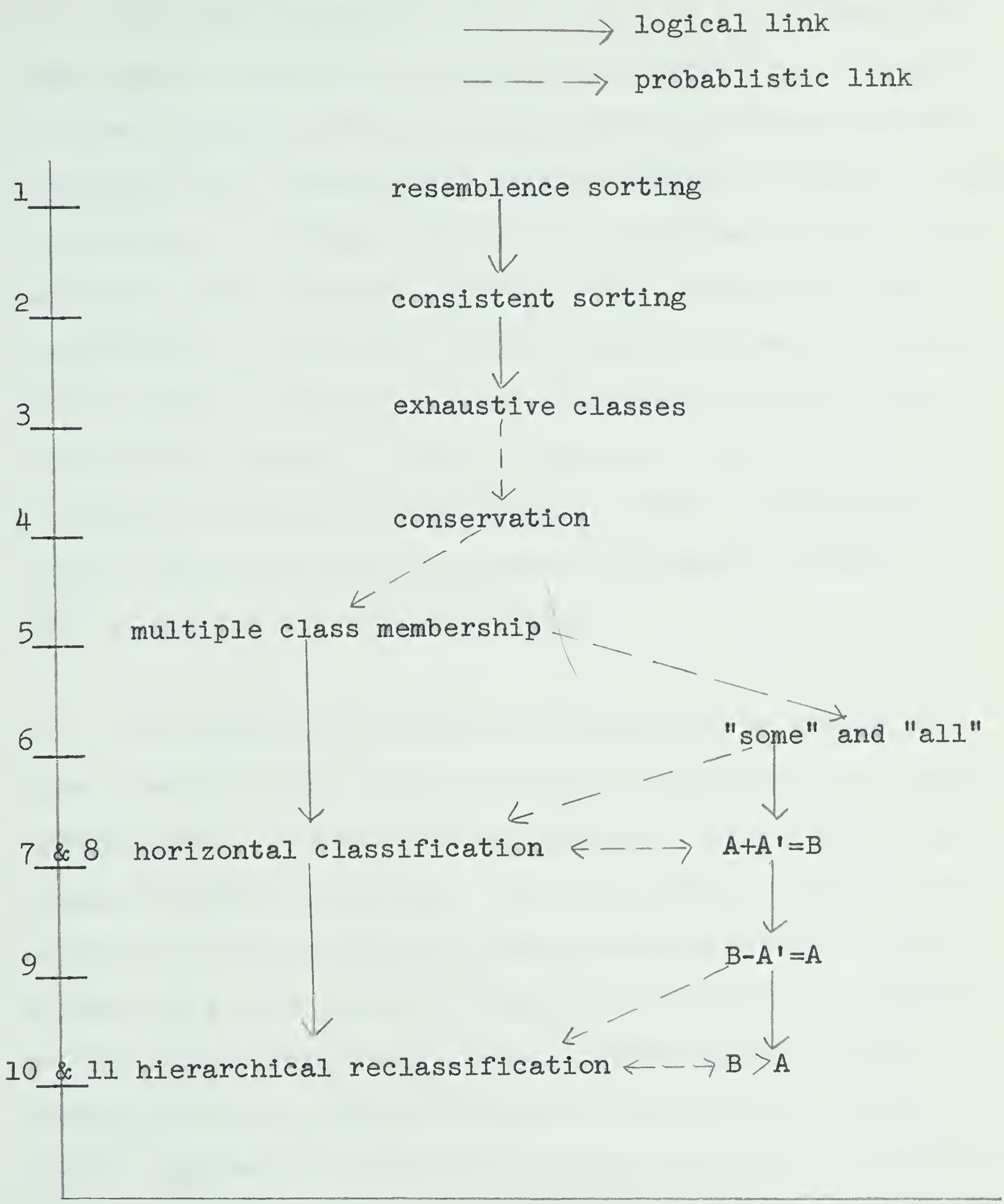

FIGURE 2

PREDICTED SEQUENCE OF DEVELOPMENT OF CLASSIFICATION SKILLS

(Kofsky, 1966, p. 194)

The realization of the existence of a complementary class occurs latter in classificatory maturation. Up until the age of eight or nine children find it difficult to even recognize a set containing a single element as forming a class. The notion of a "unique specimen" is structured by the operations of classification, but not until a systematic complementarity is involved. Piaget (1964) defines an awareness of "secondary" classes in terms of complementarity, which is the extensive aspect of such a dichotomy, and also in terms of otherness, which is its intensive aspect. Consequently, it is evident that the development of negation depends on that of the inclusion relationship.

An outgrowth of additive composition is multiplicative classification. This consists of classing each element simultaneously in terms of two additive orders and is more commonly known as a matrix. The child passes from earlier distinct classifications to multiplicative classification because of a retro-active action of the second classification on the first. This then induces an anticipatory effect allowing the two classifications to be combined. Bruner (1966) considers the matrix to be the most logically complex of classification tasks. The results of an experiment conducted by Bruner and Kenny (Bruner, 1966), in which beakers of varying heights and diameters had to be placed in a three by three matrix, indicate that the ability to replace items and reproduce a matrix increases with age. They disagree with Piaget and Inhelder in claiming that relations of similarity

are not so easily formed in early life. These relations are confounded by the "linguistic modes" used by children in describing similarities and differences. Children who employ the dimensional modes relate things to two ends of a continuum, such as fat and skinny. The global modes are completely undifferentiated. The use of the confounding modes indicates that one end of the continuum is described dimensionally and the other globally.

A strong parallel development can be drawn between classification and seriation. The serial ordering operations include the ability to generalize along a linear dimension or to arrange objects (properties) in a series. Adequate use must be made of the relational phrases "greater than" and "less than" in order to achieve this goal. The National Froebel Foundation (1961) found this is true. They claim that the real base of seriation is to see that a given item B is larger than A and smaller than C. It is their finding that true understanding of serial relations requires seeing that a series can always be given as two segments. To choose one thing to be smaller than another is to form a class of all objects smaller than the latter.

Piaget (1964) states that serial operations are an interiorized result of previous activities. This explains his assertion that seriation exists at the sensori-motor level, but that the resulting, relevant behavior is unsystematic in nature. It would appear easier for a child to

order elements rather than to classify them since serial configuration is perceptually "good form." Piaget has found this not to be the situation. The perception of the configuration is of major importance to the otongeny of serial ordering. It is only when anticipatory structures grow out of the progressive organization of actions, and when organization structures perception making perception fit its own needs, that successful sequential ordering is performed.

The two differences cited by Piaget (1964) between seriation and classification are slight; a relation can be perceived while a class as such cannot and a serial configuration is simpler and more elementary than the structure of a matrix. At Stage I the child is rarely capable of ordering items. If he can do so, it is only to the extent that he forms sub-series of elements, incapable of uniting them into a whole. This successive establishment of relations corresponds to the classificatory difficulty of coordinating intension and extension. During Stage II there is an attempt to solve the problem using preoperational methods. This haphazard effort to empirically seriate follows a trial-and-error pattern. When seriation finally is complete and the problem arises of inserting additional elements into the series, further trial-and-error tactics are exhibited and the child starts again from the beginning. Development at this phase is based on graphic anticipation as was found in the study of classification. This level corresponds to the incomplete coordination of intension

previously discussed. Finally, when operational seriation is attained, the child procedes systematically to look for the smallest among those remaining after first looking for the smallest element. Here we see reflected the use of ascending and descending methods necessary to successful classification. Also at Stage III, new elements can be correctly inserted into an existing series without haphazard quessing.

Elkind (1964) confirmed Piaget's results concerning the development of seriation ability. In his replication study he distinguished the same three stages. First, he found that children formed a general impression of the series as a global figure in which the part was undifferentiated from the whole. Because of the perceptual orientation of the child's logic at this stage, he is unable to order pairs of relations. The Stage I child mistakes his abscence of a visual impression for the abscence of a real object and will announce that the task you have set for him cannot be done. Elkind attributes the trial-and-error method used by the Stage II child to the imagined character of the representation he has formed. This also explains the observed inability to insert a second set of elements within a completed series. This child sees the objects as an unordered, completed whole and consequently cannot insert new elements. The typical child of the third stage who performs without error does so because he can attribute the results of his mental operations to the perceived elements. His coordinated acts have become internalized and now form logical operations.

V. CONNECTION BETWEEN LOGICAL PROCESSES AND NUMERICAL CONCEPTS

Numerical thinking is closely related to logical thinking and it seems likely that each depends on the other (Isaacs, 1960). When one considers that classification is based on similarity and seriation on cumulative differences, it becomes apparent that the concept of number, founded on cardinal and ordinal abstractions, is a synthesis of classification (hierarchies of wider classes) and seriation (arrangement in a graduated order). With this in mind Dodwell (1962) hypothesized that some understanding of class composition is necessary for dealing "operationally" (consistenly) with number. His sample consisted of sixty children from knidergarten through third grade, all from five to eight years of age. Their I.Q.s ranged from eighty-one to one hundred eighteen. After investigation Dodwell concluded that an understanding of class composition and hierarchical classification develops "... independently of an understanding of the concept of cardinal number (Dodwell, 1962)." This is not to say that classification has no connection with cardination; as Piaget has demonstrated classification and seriation develop concurrently with cardinal and ordinal number (Piaget, 1965). Piaget asserts that when a child applies an operational system of classification and seriation to sets defined by the qualities of their elements, he is compelled to consider separately classes, which depend on the qualitative equivalence of their elements, and asymmetrical

relations, which express the seriable differences present, (Piaget, 1965). Consequently, classes are non-seriated numbers and numbers are seriated classes. Number is a synthesis of classification and seriation; number is not just a class, because to obtain a cardinal value elements must be ordered to be counted.

The notion of quantity, to the younger child, depends less on number than on the global appearance of the set and the space occupied by it. Therefore, the child does not use a one to one correspondence as a quantifying basis. A quantifying correspondence presupposes the equalization of difference and perceptual correspondence. Great care must be taken not to consider a mere one for one exchange as a guarantee that the child sees two equivalent sets as being bound by the same cardinal number; this recalls the confusion of the Stage I child over extension and intension of a class. Consequently the child who cannot classify experiences great difficulty with integers, which are essentially formed by a one to one correspondence. Equating the growth of number concepts with the development of classification, it is found that quantification by gross quantity occurs as a global evaluation (Stage I), and quantification resulting from qualitative correspondence or numerical correspondence is a function of intensive evaluation or extensive evaluation, respectively (Stage II). (Piaget, 1965).

A similarity which has become numerical, where by items

are distinguished one from another by order, is termed ordinal correspondence. This correspondence is acquired on the operational level, owing to its fusion with cardination. It is only when each element is combined with the preceding ones that its position can be determined just as it is only their positions that differentiate the units, which in other respects are all equivalent (Piaget, 1965).

Piaget (1965) has distinguished three stages in the increasing ability of children to handle cardinality and ordinality. At the first stage cardinal evaluation is a global judgment and depends on the configuration of the set, the space it occupies, and the spread of the elements. Seriation is only the juxtaposition of terms with no law governing the order. In summation, the Stage I child in his efforts to seriate, disregards sets, and in his attempts to evaluate totalities he disregards order. During the second stage seriation is done in terms of classes and the sets they define. The cardinal whole exists only so long as it is perceived as such. The reversible operations characteristic of the final stage imply how it is possible to set the series out in either direction. The concept of the equivalence of elements leads to the construction of logical classes. Likewise, non-equivalence leads to asymmetrical relations.

Simple classification and straight seriation are not the only logical processes involved in number conceptualization. Additive and multiplicative operations are implied also,

since number is an additive union of units and a one to one correspondence between sets entails multiplication of classes. If we regard the extension of classes as being inseparable from their comprehension, it becomes clear that classes and number have a common basis - the additive operation of bringing together scattered elements into a whole or dividing wholes into parts. Often the child cannot see the whole as the result of an additive composition of parts, and therefore as soon as he envisages one part separately the whole as such is destroyed. The addition of classes seems then to imply logical multiplication, i.e. each element belonging to a system of added classes necessarily belongs to two classes simultaneously, all elements of either part possessing the attributes of that part and of the whole. To transform class F and F_1 into numbers their terms must be regarded as equivalent from all points of view concurrently, and the equivalent terms must remain distinct. The only difference between number and class is that classes are qualified, united by virtue of common qualities while in number the parts remain homogenous (Piaget, 1965).

The additive and multiplicative compositions of asymmetrical relations make measurement possible. Measurement is impossible without conservation of the qualities to be measured; if units do notremain invariant they cannot be composed. The composition of conserved units and the introduction of a system of equivalence between these compositions constitute the logic of measuring. Once the child

becomes capable of the numerical composition involving relations, he also becomes capable of elementary operations involving these relations.

Number work in the first grade is primarily concerned with quantitative concepts. In the abstraction of quantitative invariance resemblences yield groups and asymmetrical differences indicate quantification. Thus, seriation is the basis upon which the primitive quantitative relations of "more" or "less" are structured. Classification furnishes the ground upon which the vital concept of one to one correspondence is built. Classification and seriation cannot therefore be divorced from mathematical achievement, because it is precisely these abilities which must be developed before qualitative (intensive in nature) and numerical (extensive in nature) correspondences can be made.

CHAPTER III

THE EXPERIMENTAL DESIGN

This study was concerned with determining the efficiency of logical processes as predictors of mathematical achievement. The sample consisted of one hundred thirteen pupils who were members of five randomly selected grade one classrooms. To determine the relationship of logical processess to mathematical achievement two testing experiences were constructed.

The first testing, a type of individual assessment, demanded essentially non-verbal responses from the children on four tasks which were adaptations of those discussed by Piaget (1964). The first two tasks involved simple classification and additive composition of classes (class inclusion). The third and fourth tasks dealt with multiple seriation and simple sequential ordering, respectively. Responses were scored at the completion of each activity.

The above testing was followed a week later by the administration of the Seeing Through Arithmetic Test (S.T.A.T.), given in accordance with the specifications of the Edmonton Public School Board. This test requires a written response to orally and visually presented items. All classroom teachers gave the test themselves, presenting Part 1 in the morning and Part 2 in the afternoon of the same day.

This chapter contains more detailed information about the sample, testing apparatus, and statistical procedures followed.

I. THE SAMPLE

The sample was composed of one hundred thirteen students, members of the five grade one classes allocated by the Edmonton Public School Board. The age range of the sample was from seventy-nine to eighty-eight months. The mean chronological age was 82.7 months, and the standard deviation was 3.6. The mean age is lower than the approximate age for the onset of the concrete operational stage which Piaget suggests. Consequently, there may be fewer children in this sample who are performing at the concrete operational stage and are able to coordinate the processes of classification and seriation than one might expect to find in other first grade classes.

The Intelligence Quotients varied from fifty-seven to one hundred fifty-three as measured by the Detroit Beginners First Grade Intelligence Test. The mean I.Q. for the sample was 98.0 and the standard deviation was 17.8. The distribution of Intelligence Quotients is shown in FIGURE 3.

No attempt was made to select the sample with regard to socio-economic factors. After making individual approximations using the "Blishen Occupational Class Scale" it was found that sixty-one of the children came from middle class

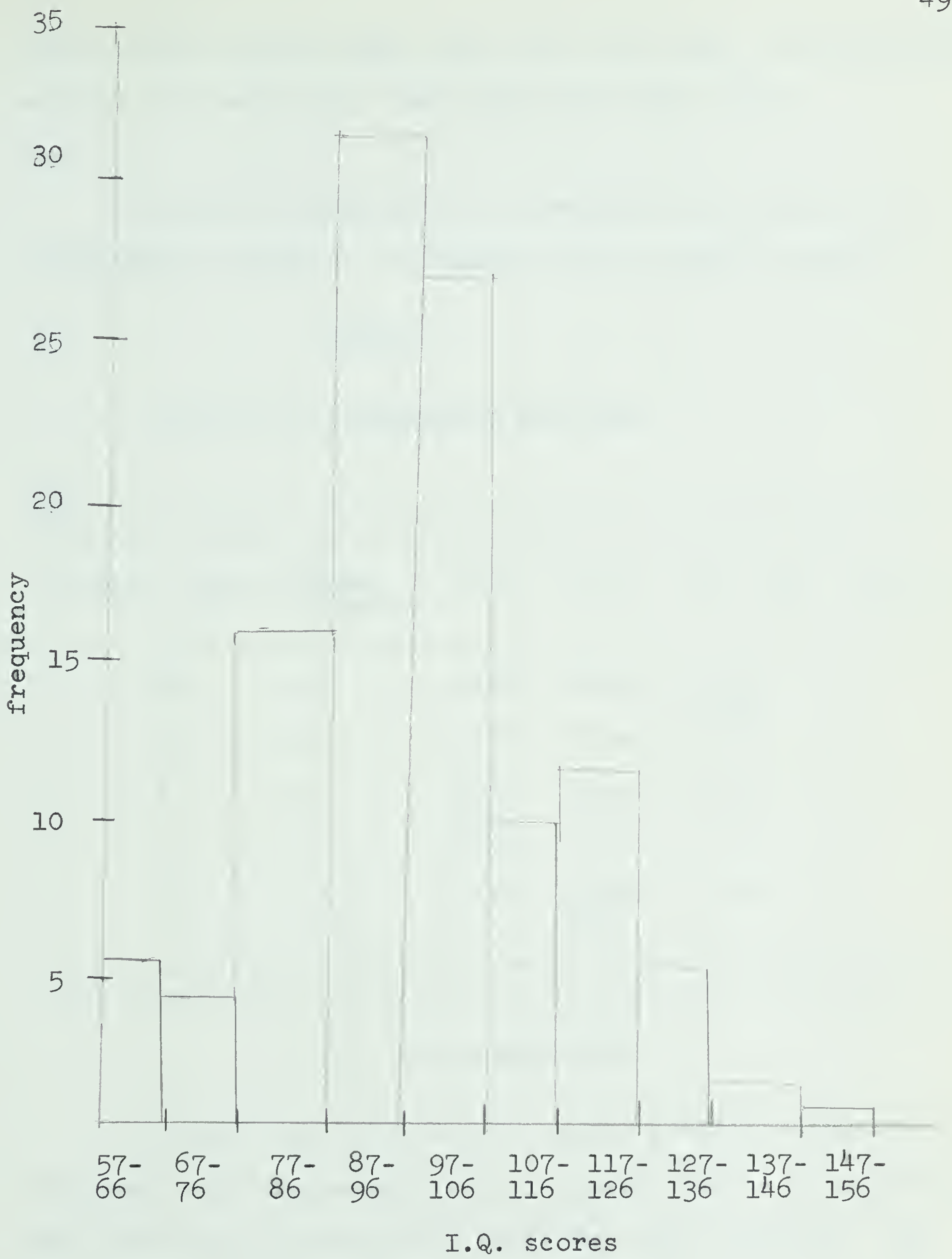

FIGURE 3

DISTRIBUTION OF THE INTELLIGENCE QUOTIENTS OF 113 FIRST GRADE PUPILS

homes, while twenty-eight came from the higher socio-economic strata and twenty-four came from the lower class.

A brief history was also compiled for each of the cooperating teachers. A summary may be found in TABLE I.

TABLE I

HISTORY OF COOPERATING TEACHERS

Teacher	Years taught total	Years taught Edmonton	Where	Level	Training	Years on S.T.A.
1	28	22	Alta.	Elem.	B. Ed. 5 yrs.	3
2	21	16	Alta.	Elem.	4 yrs.	3
3	8	4	Can. U.S.	Elem.	5 yrs.	2
4	5	5	Alta.	Elem.	4 yrs.	3
5	33	15	Alta.	Elem. Sec.	B. Ed.	2

II. THE INSTRUMENT

As mentioned earlier, the major portion of the individual tests was adapted from Piaget's work. In addition, one task was devised specifically for this research. The individual testing followed this format:

Classification items

Task I - simple classification

Task II - additive composition of classes

Seriation items

Task III - multiple seriation

Task IV - simple seriation

Items on Tasks I, III, and IV required non-verbal responses while items on Task II required only a yes or no answer.

The materials were set up on large tables and were immediately evident to the children as they entered the room. Conversation was begun upon enterance, and a sincere effort was made to replicate the dialogue on every ocassion. A prepared dialogue sequence was recorded on an index card, and it was periodically referred to during the testing period. The order in which the tasks were presented was never varied. The task requiring the most time was presented first while interest was still fresh. The task demanding the most physical activity was presented last.

The objects used to determine classificatory ability consisted of two groups of materials. The first task, simple classification, allowed the child to demonstrate the criterion he was able to use in order to form simple groups. The test provided the child with numerous objects to be sorted into piles in any manner he saw fit. The two bases for classification which the material depicted were form and color. The objects used for this task included thirty flat, geometrical construction paper shapes. These shapes were either squares, triangles, half-rings, circle, or rectangles.

Each shape appeared in three different colors. The array was composed of ten gray, ten yellow, and ten blue shapes. Of the ten gray shapes three were half-rings, three were triangles, one was a square, another was a circle, and two were rectangles. Among the yellow shapes there was one half-ring, three triangles, two squares, one circle, and three rectangles. The blue shapes included two half-rings, three triangles, three squares, one circle, and one rectangle. All these pieces were distributed on the table in a random fashion (FIGURE 4). All of the squares had four and one-quarter inch sides, and all the circles had a five inch diameter. The triangles were all isoceles with three and three-quarter inch sides and six inch base. The rectangles measured one and three-quarter inches by six and one-quarter inches, and the half-rings had three and one-quarter inch diameters and were one-half inch thick.

FIGURE 4

SIMPLE CLASSIFICATION - TASK I

The second task dealt with additive composition of classes. This task was designed to assess whether the child could distinguish between the term "all" as applied to the perceptual whole while the structural whole remained visible. The materials used in this second task were nine construction paper shapes of two forms and two colors. There were two blue squares and two orange squares, with three and five-eights inch sides. There were also five orange circles with a five inch diameter (FIGURE 5). An array was displayed before the child, and he was asked two questions pertaining to the shape and color of the objects. One of the questions was about the total group of squares. The other question was about the shape of all blue pieces of paper.

FIGURE 5

CLASS INCLUSION - TASK II

The objects used to determine seriation ability also consisted of two sets of materials. The multiple seriation task was designed to serve as a means for establishing whether the child could usefully coordinate order relations along two dimensions. The set of materials used consisted of twenty-four brightly colored pictures of Christmas ornaments, clipped from a magazine. The pictures were pasted onto two pieces of black construction paper; they were arranged in four rows and six columns (FIGURE 6). The child was asked to locate two pictures after being given their horizontal and vertical coordinates.

FIGURE 6

MULTIPLE SERIATION - TASK III

The final task involved simple seriation. This task allowed the child to demonstrate his ability to sequentially order elements along a linear dimension. An assortment of items, each obviously varying from the other, was to be put into proper sequence. These materials consisted of a set of ten cans ranging in height from one and seven-eighths inches to fifteen inches and covered with tin foil to minimize all perceptual differences other than height (FIGURE 7). If the ordering was successfully accomplished, an additional collection was given the child to be correctly placed within the established sequence. A set of five intermediate lengths formed this collection (FIGURE 8).

III. OTHER MEASURES

At the beginning of September the children took the Detroit Beginners First Grade Intelligence Test. The Intelligence Quotients given by this test were then entered on each student's cumulative record card. It was from these cards that the Intelligence Quotient was obtained. Other personal facts such as name, sex, birth-date, age in years and months, grade level, date, and the occupation of both parents were also secured from the files.

A subjective ranking was also collected from each cooperating teacher, which indicated whether she thought each student to be a high, medium, or low achiever in mathematics.

FIGURE 7

SIMPLE SERIATION - TASK IV

FIGURE 8

INTERMEDIATE SET - TASK IV

In addition use was made of the S.T.A.T. This test consists of sixty items, divided into two sections; one deals with mathematical concepts and applications, the other with computational skills. Part 1 has twenty-four questions and Part 2 has thirty-six. It is a timed test. The first section proceeds from question to question as the teacher reads the direction for each item orally and the children respond. The second section allows the children to work on their own after receiving the appropriate instructions.

IV. THE TESTING PROGRAM AND METHOD OF SCORING

THE INDIVIDUAL TESTING SECTION

Each child underwent a testing period of ten minutes to determine

1. his capability of perceiving more than one criterion for classificatory purposes
2. his ability to distinguish between the perceptual whole and its components as indicated by the terms "all" and "some"
3. his capacity to consider two ordered relations simultaneously
4. his ability to put elements into sequential order while considering only one dimension of these elements.

Since the success of cummunicating with the child is of importance, a detailed account of the testing session is given in Appendix A.

Each child was tested individually in a private room in the school. The children were told that they were going to play a game. The teacher then instructed the children as to how they were to leave the room and where they were to go when it was their turn to play the game.

SCORING

The data were compiled on one sheet to permit personal information and raw scores to be recorded simultaneously. The personal facts collected were extracted from the cumulative record files kept by the school.

Each of the four tasks was scored individually. On each task the highest possible score was two; the lowest was zero. The tasks concerning simple classification and simple seriation were scored according to the stage at which the child performed. The child was assumed to be performing at the Stage III level if he was able to group the objects in two different ways, according to their form and their color. Performance at this level earned the child two points. The less adept children who performed at a Stage II level received one point. Their behavior was characterized by an ability to classify, but only according to one criterion. Stage I children received no credit. These children could not group the objects in any logical way. When asked to do so they merely did not respond or only juxtaposed pieces in a random manner. When investigating seriation behavior the same point system was applied. Stage III performance was

characterized by a successful ordering of the elements without resorting to a trial-and-error method. At this stage the second set would also be correctly dispersed amongst the original cans. When a child behaves at a Stage II level he is able to order the elements. But this child would use a trial-and-error technique to do so . A Stage II child would also handle the intermediate set satisfactorily. At Stage I the child would be unable to work with either the full set or the intermediate set.

The tasks dealing with class inclusion and multiple seriation were scored according to the number of questions answered correctly. Each of these tasks involved answering two questions. One point was given for each correct answer.

Individual scores were later combined. If a child received a score of three or better on Tasks I and II combined, it was assumed he could classify. If a child received a score of three or better on Tasks III and IV combined, it was assumed he could seriate. It was assumed that the child could coordinate the operations of classification and seriation if his cumulative score was seven or better. This criterion score was based on the assumption that some additional process, other than just the logic of classification and seriation, was involved in the coordination of these abilities.

Scoring for the S.T.A.T. was done on a point basis,

each correct answer receiving a score of one. A perfect paper had a total score of sixty. The total S.T.A.T. score and each separate score for Part 1 and Part 2 was recorded. Certain items on the S.T.A.T. were designated as demanding either classificatory schema or sequential ordering ability. Credit was given for the successful completion of these items when seventy-five per cent of the questions were answered correctly. Of the five classificatory items on the S.T.A.T., four had to be successfully completed for the child to receive credit for possessing classificatory logic. Of the seven seriational items on the S.T.A.T., five had to be successfully completed for the child to receive credit for possessing sequential ordering ability.

Each classroom teacher was asked to place her individual students in a high, medium, or low group according to their arithmetic ability as interpreted by the teacher herself. Each child's particular ranking was entered on his data sheet.

V. THE PILOT STUDY

A pilot study was carried out in February 1968 using nineteen children who did not form part of the sample. The pilot work was to assess:

1. whether there was a sufficient number of first grade students who performed at various developmental levels
2. the effectiveness and efficiency of the individual testing apparatus
3. whether there was some indication of a positive

relationship between an individual's performance on the Piaget-like tasks and his teacher's subjective evaluation of his mathematical achievement.

On the basis of this preliminary work it was decided that the use of only first grade pupils would provide a sufficient sampling of the three Piagetian stages. The pilot work also showed that the method of questioning and the materials used were satisfactorily handled by first grade pupils. There was also found to be some positive relationship between an individual's performance on the individual tasks and his teacher's evaluation of his arithmetic ability.

VI. THE ANALYSIS TO BE USED

The data were analysed using multiple linear regression. This analysis lends itself very well to investigations in which the hypothesis can be stated in terms of a question expressing a functional relationship. It is also frequently used when performance is designated in terms of the additive combination of group membership vectors. It was for these reasons that this type of analysis was followed.

Decisions to accept or reject the null hypotheses were made at the .05 level of significance. The variable of I.Q. was incorporated into the analysis in an attempt to separate its effects from those of classificatory and seriational ability.

CHAPTER IV

RESULTS OF THE INVESTIGATION

It was the object of this investigation to determine the relationship between mathematical achievement and logical thought processes in grade one children. A sample of fifty-five girls and fifty-eight boys was selected from five elementary schools in the Edmonton Public School System. Two tests were administered to the children in the sample. A logic test was administered individually during the final week of May, 1968. One week later the classroom teachers administered the Seeing Through Arithmetic Test (S.T.A.T.).

A summary and analysis of the logic test and the mathematical achievement test are given in this chapter. Inter-correlations between these measures and the variables of I.Q., age, sex, and arithmetic ability are also reported and discussed.

I. NATURE OF THE SAMPLE

The sample consisted of one hundred thirteen first grade pupils who were randomly selected. The mean I.Q. for the sample was 98.0, and the standard deviation was 17.8. The mean chronological age was 82.7 months, and the standard deviation was 3.6. The mean age is lower than the approximate age for the onset of the concrete operational stage which Piaget suggests. Consequently, there may be fewer children in this sample who are performing at the concrete

operational stage and are able to coordinate the processes of classification and seriation than one might expect to find in other first grade classes. Each subject was assigned a number from the "Blishen Occupational Class Scale." The number assigned was dependent upon the occupation of the subject's father as recorded on the school record file. Sixty-one of the children came from middle class homes. Twenty-eight of them came from upper class homes and twenty-four from lower class homes.

Each teacher was asked to rank her students as having either high, medium, or low arithmetic ability. The distribution of pupils among these ranks is shown in the table below.

TABLE II

TEACHERS' SUBJECTIVE EVALUATION OF PUPILS' ARITHMETIC ABILITY

Ability group	Number in group	Per cent of sample in group
high	36	31.9
medium	46	40.7
low	31	27.4

II. THE TEST OF LOGIC

This test consisted of four tasks. The first two tasks dealt with classification, and the other two tasks

dealt with seriation.

The first part of the logic test dealt with classificatory logic. The first task was an exercise in simple classification. The child had to group objects according to their form and color, but not simultaneously. The materials used were of five different shapes and three different colors. If a correct grouping was made the child received a score of two. If the child fluctuated between grouping by color or form or if he used only one of these criteria, a score of one was assigned. A child who was unable to classify in any of these ways received a zero score. Seventy-one of the children in the sample grouped according to only one characteristic. Of these children four boys and one girl chose color as the classificatory base. The remaining sixty-six chose form as the basis of classification.

The other classification task was designed to assess the child's ability to answer questions relating to the membership of a group. An array of blue squares and orange circles and squares was shown to the child. He was then asked two questions. One question was about the total group of squares, and the other question was about the shape of all the blue objects. A score of one was given for each correct answer.

These scores were then combined to yield a total classificatory score. If this score was three or better it was assumed the child could classify. Seventy-four children of

the one hundred thirteen in the sample successfully completed both tasks involving classification.

The second part of the logic test dealing with sequential ordering also consisted of two tasks. One task served as a means for establishing whether the child did coordinate order relations along two dimensions. Pictures arranged on construction paper in rows and columns were shown to the child, and he was asked to locate two specific pictures after being given their location in terms of rows and columns. Each correct response received a score of one.

The other task involved simple seriation. The child was asked to order cans along a linear dimension. He was told where to place the shortest and tallest cans and was then left to put the rest of the cans in the correct order. If he was successful, the child was given additional cans to place in the established sequence. Performance was scored according to the operational level at which the child was functioning. The highest level of performance was characterized by the successful ordering of all cans without the use of trial-and-error. This solution received a score of two. If the child was able to order the cans but had to use trial-and-error, a score of one was given. If the child failed to order the elements he was given a score of zero.

These scores were then combined to yield a total seriation score. If this score was three or better, it was assumed

that the child could seriate. Fifty of the one hundred thirteen children in the sample were successful on both seriation tasks.

Finally, all scores were combined. It was assumed that some additional process was needed to coordinate the processes of classification and seriation other than the two abilities themselves. If a score of seven or better was achieved, it was assumed that the child was capable of coordinating the actions of classifying and ordering. TABLE III indicates that only twenty-two children were able to do this. The majority of the first graders in this sample found it difficult to coordinate classification and seriation. However, over half the grade one children in this sample were able to classify and almost half were able to seriate. Additional information about the logic test scores is given in TABLE IV.

The inter-correlation between the classification and seriation parts of the logic test was .16. This correlation is not significant at the .05 level ($r \geq .195$).

III. THE MATHEMATICAL ACHIEVEMENT TEST

The S.T.A.T. was given by the classroom teachers as the culmination of the year's course of study in the Seeing Through Arithmetic program. The S.T.A.T. consists of sixty items divided into two sections. One section of twenty-four items deals with mathematical concepts and applications.

TABLE III

DIFFICULTY OF USING LOGICAL PROCESSES EXPRESSED AS THE PER CENT OF CHILDREN EMPLOYING SUCH PROCESSES

Ability to	Number possessing this ability	Per cent possessing this ability
classify	74	65.5
seriate	50	44.3
coordinate both	22	19.5

TABLE IV

ANALYSIS OF LOGIC TEST SCORES IN TERMS OF MEAN, MEDIAN, AND MODE

	Mean	Median	Mode
classification	2.77	3.18	3
seriation	2.25	1.71	2
both	5.15	4.61	6

The other section tests immediate recall of basic addition and subtraction facts and contains thirty-six items.

The scores on the S.T.A.T. for this sample ranged from fifteen to sixty. Two children received a perfect score of sixty. The distribution shown in FIGURE 9 is negatively skewed. This indicates that the exam was relatively easy for the majority of the students. Therefore it is reasonable to assume that this instrument does measure mathematical achievement in terms of the S.T.A. course.

The frequency distribution of scores for Part 1, Mathematical Concepts and Applications is shown in FIGURE 10. These scores ranged from two to twenty-four. Scores on this sub-test tend to cluster around the upper intervals of the scale.

The distribution of scores on Part 2, Computational Skills is shown in FIGURE 11. Almost half of the pupils in the sample scored between thrity-four and thirty-six on this sub-test. The maximum score of this sub-test was thirty-six.

Additional information concerning the results of the S.T.A.T. is given in TABLE V, page 72. The mean of the first part with twenty-four items was 17.9, and the standard deviation was 4.1. Part 2 involved thirty-six items and its mean was 28.7 with a standard deviation of 9.0. The mean of the total test of sixty items was 46.7 with a standard deviation of 11.8.

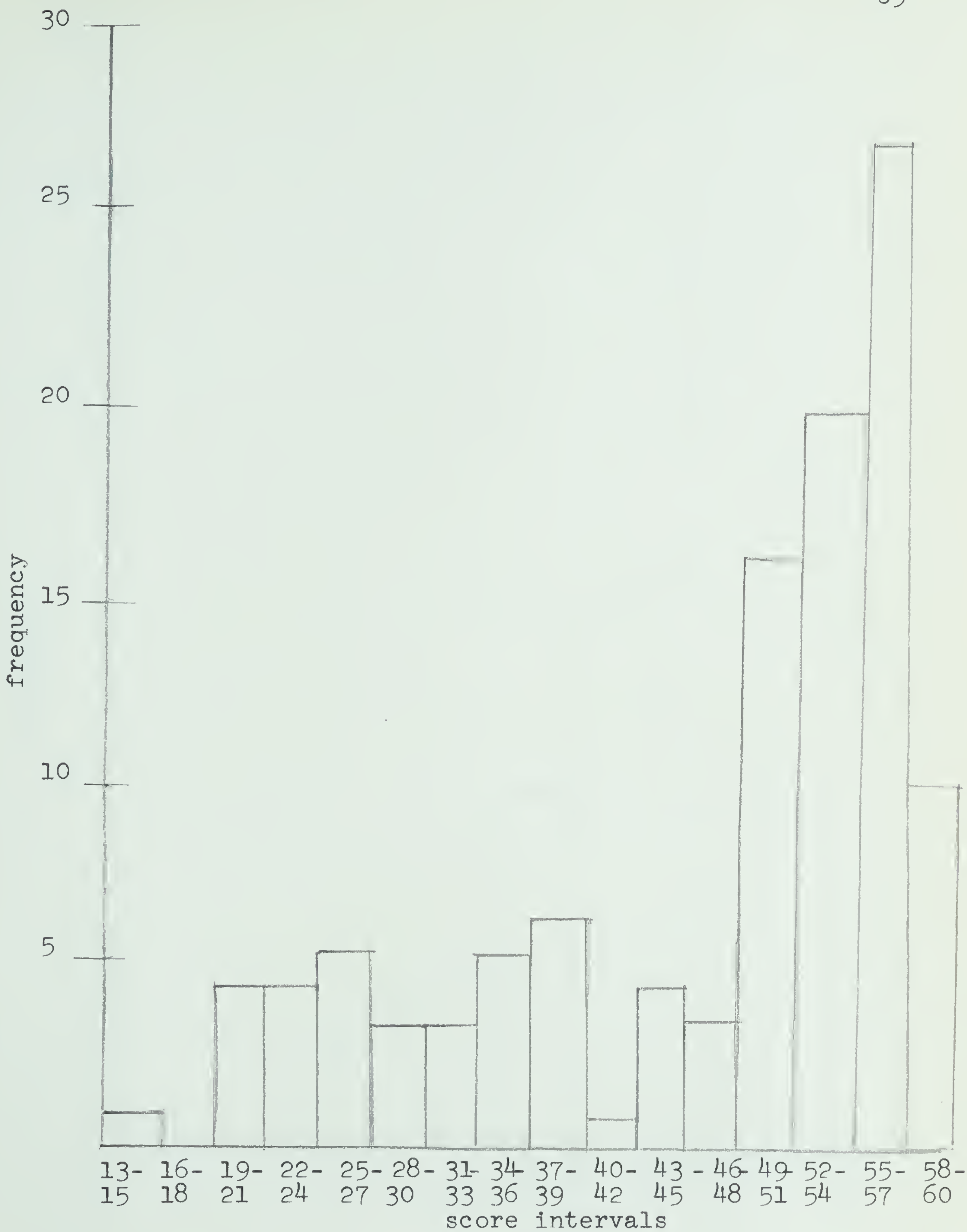

FIGURE 9

A FREQUENCY DISTRIBUTION OF TOTAL S.T.A.T. SCORES

(n=113)

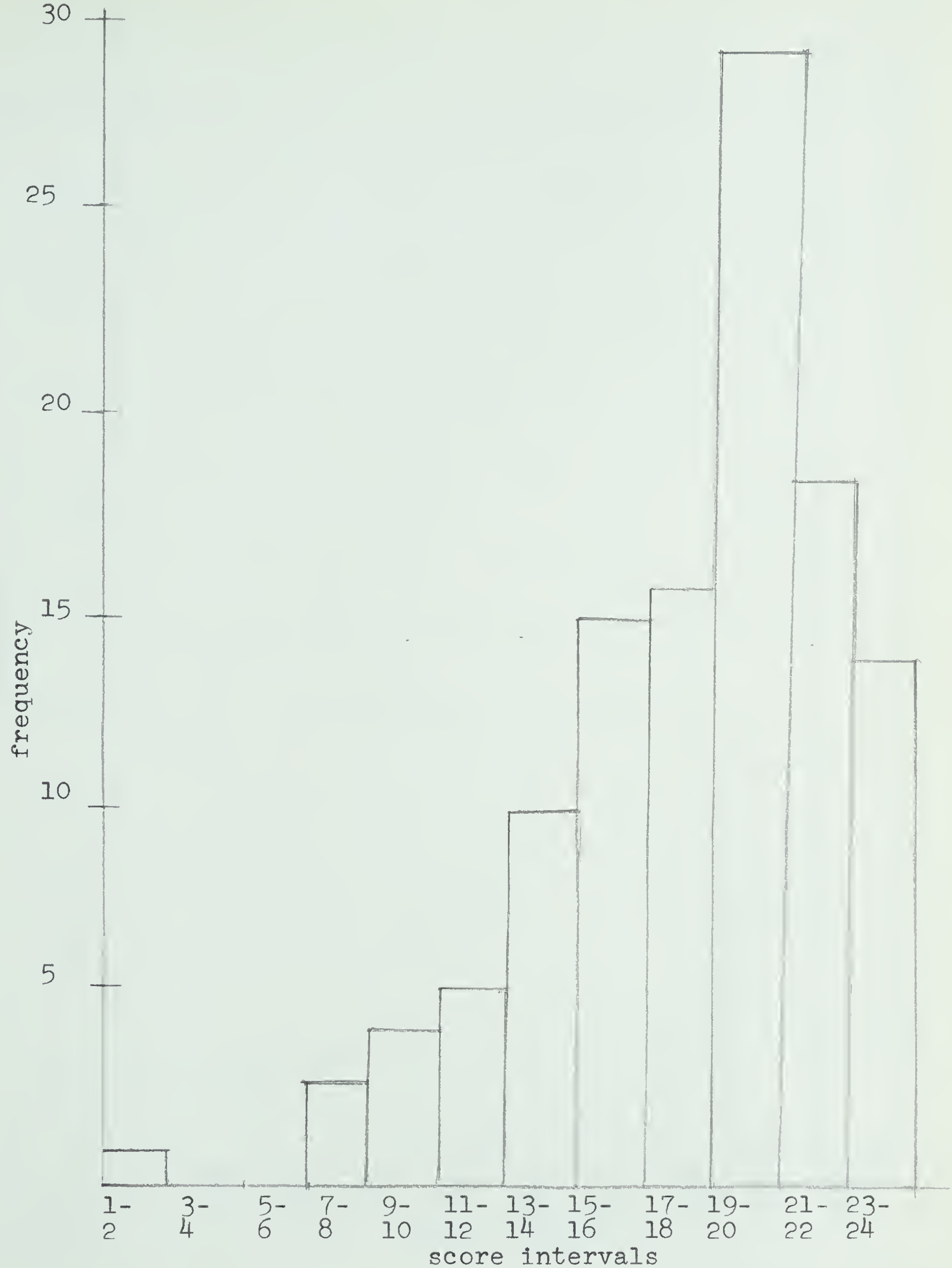

FIGURE 10

A FREQUENCY DISTRIBUTION OF THE PART 1 S.T.A.T. SCORES

(n=113)

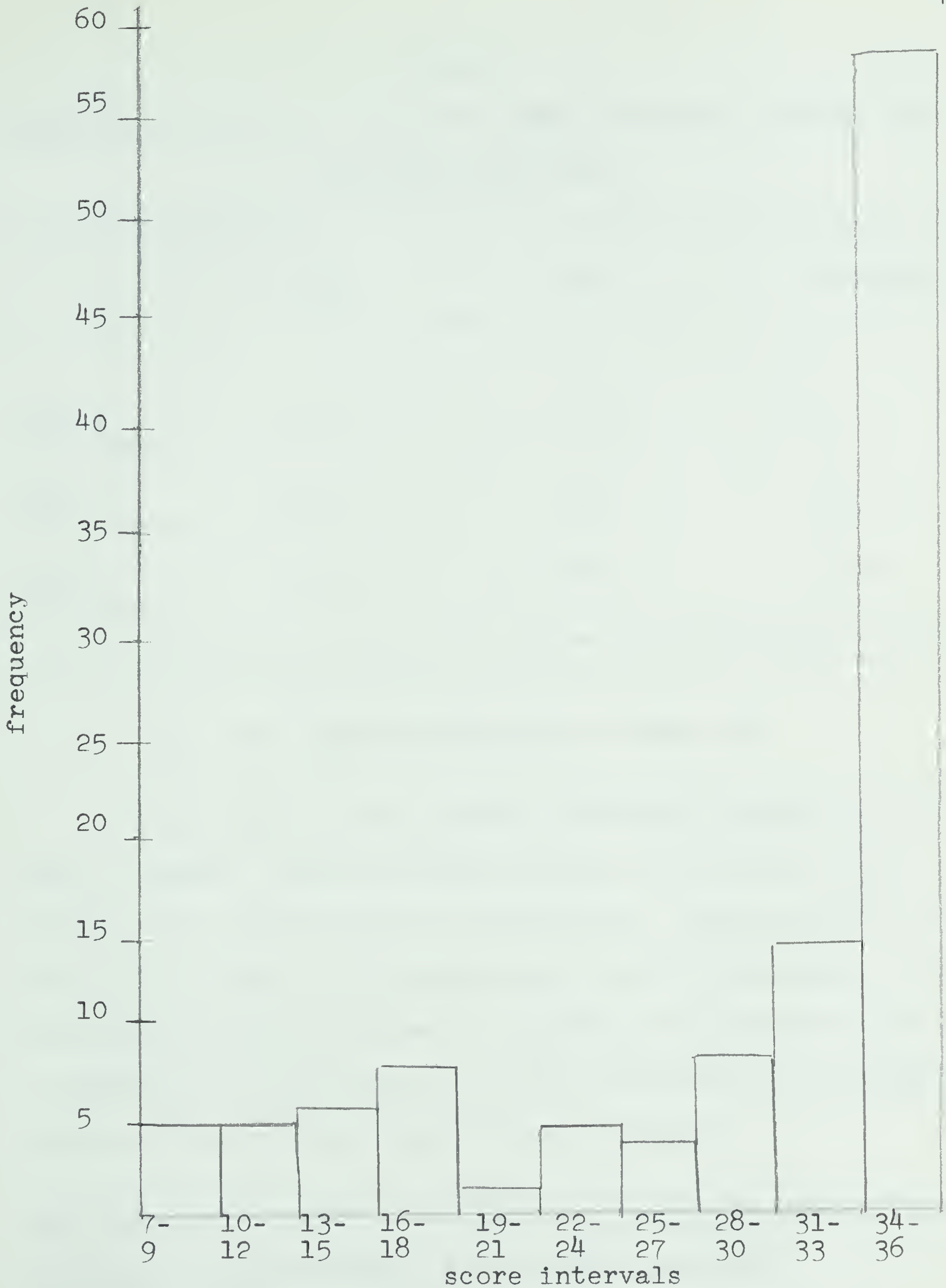

FIGURE 11

A FREQUENCY DISTRIBUTION OF THE PART 2 S.T.A.T. SCORES

(n=113)

TABLE V

ANALYSIS OF S.T.A.T. SCORES IN TERMS OF RANGES, MEANS, AND STANDARD DEVIATIONS

	Range	Mean	Standard deviation
Part 1 (24 items)	2-24	17.9	4.1
Part 2 (36 items)	8-36	28.7	9.0
total (60 items)	15-60	46.8	11.8

IV. THE INTERCORRELATIONS OF VARIABLES

This analysis was carried out using the Reg 200 computer program. Pearson product-moment correlation coefficients were computed for all variables. Those correlations which were found to be significant will be discussed. With one hundred eleven degrees of freedom on a two-tailed test, a correlation coefficient of .195 is required to be significantly different from zero at the .05 level.

THE RELATION OF LOGICAL PROCESSES TO VARIABLES OTHER THAN MATHEMATICAL ACHIEVEMENT AND TEACHERS' RANKINGS

The correlation coefficients of the relationship between logical processes, Intelligence Quotient, age, and sex are given in TABLE VI. Logical thought processes do not

correlate significantly with age or sex. They do correlate significantly with I.Q. Sex cannot be used to predict what logical thought processes the child possesses. The lack of correlation between logic and age is surprising. Piaget has suggested that the development of logical thought is partly due to maturation. However, a cursory examination of the ages of the children who could coordinate the processes of classification and seriation showed that sixteen of the twenty-two in this group were at least eighty-one months of age. Of those remaining six pupils who were credited with the ability to coordinate seriation and classification only one had an I.Q. below 109. The relationships discussed can be seen in FIGURE 12.

TABLE VI

CORRELATIONS OF LOGIC TEST SCORES WITH I.Q., AGE, AND SEX

Logic test scores	I.Q.	Age	Sex
total score	.241*	.172	.148
classification score	.331**	.051	.131
seriation score	.203*	.008	.191

* significant at the .05 level (r $\geqq$ .195)

**significant at the .01 level (r $\geqq$ .254)

THE RELATION OF TEACHERS RANKINGS TO VARIABLES OTHER THAN LOGICAL PROCESSES OR MATHEMATICAL ACHIEVEMENT

The coefficients of correlation for the relationship

between teachers' ranking of children's arithmetic ability and the variables of Intelligence Quotient, age, and sex are presented in TABLE VII. Teachers' high evaluations are positively and significantly correlated with intelligence scores. Teachers' medium and low evaluations are negatively correlated with intelligence. This finding indicates that teachers are more successful identifying pupils of high mathematical ability than in locating pupils with only a medium or low degree of mathematical ability.

A further analysis of the relationship is given in FIGURE 13, page 76. Of the twenty-six pupils whose Intelligence Quotient is one hundred ten or above only two were designated by their teachers as having a low arithmetic ability. Fourteen children had Intelligence Quotients below eighty. Only one of these students was ranked as having a high arithmetic ability.

TABLE VII

CORRELATIONS OF TEACHERS' SUBJECTIVE RANKING WITH I.Q., AGE, AND SEX

Ranking	I.Q.	Age	Sex
high	.461**	.108	.094
medium	-.064	-.055	.130
low	-.410**	-.051	-.242*

* significant at the .05 level ($r \geqq .195$)

**significant at the .01 level ($r \geqq .254$)

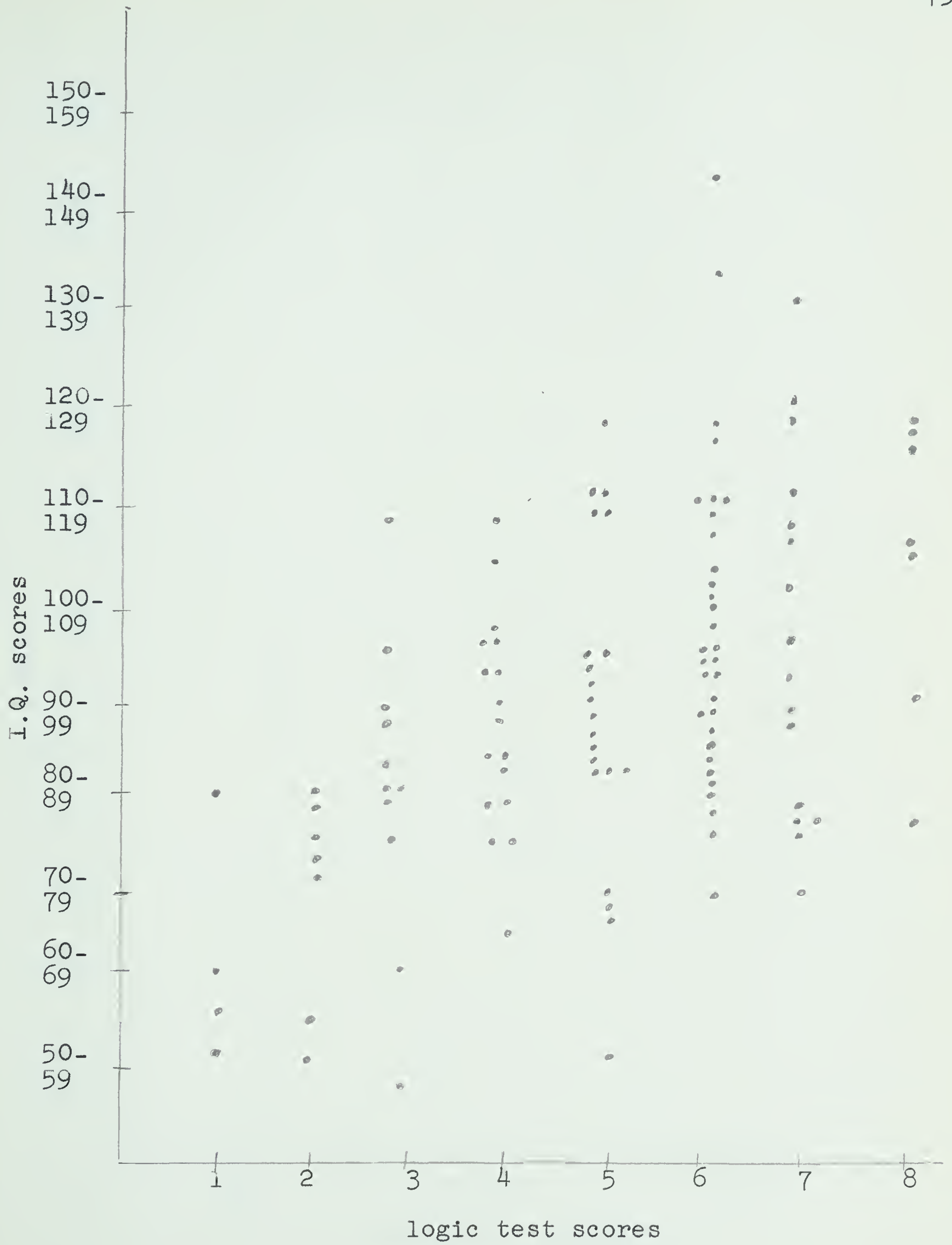

FIGURE 12

SCATTER DIAGRAM OF THE RELATIONSHIP BETWEEN LOGICAL PROCESSES

AND INTELLIGENCE QUOTIENTS
(n=113)

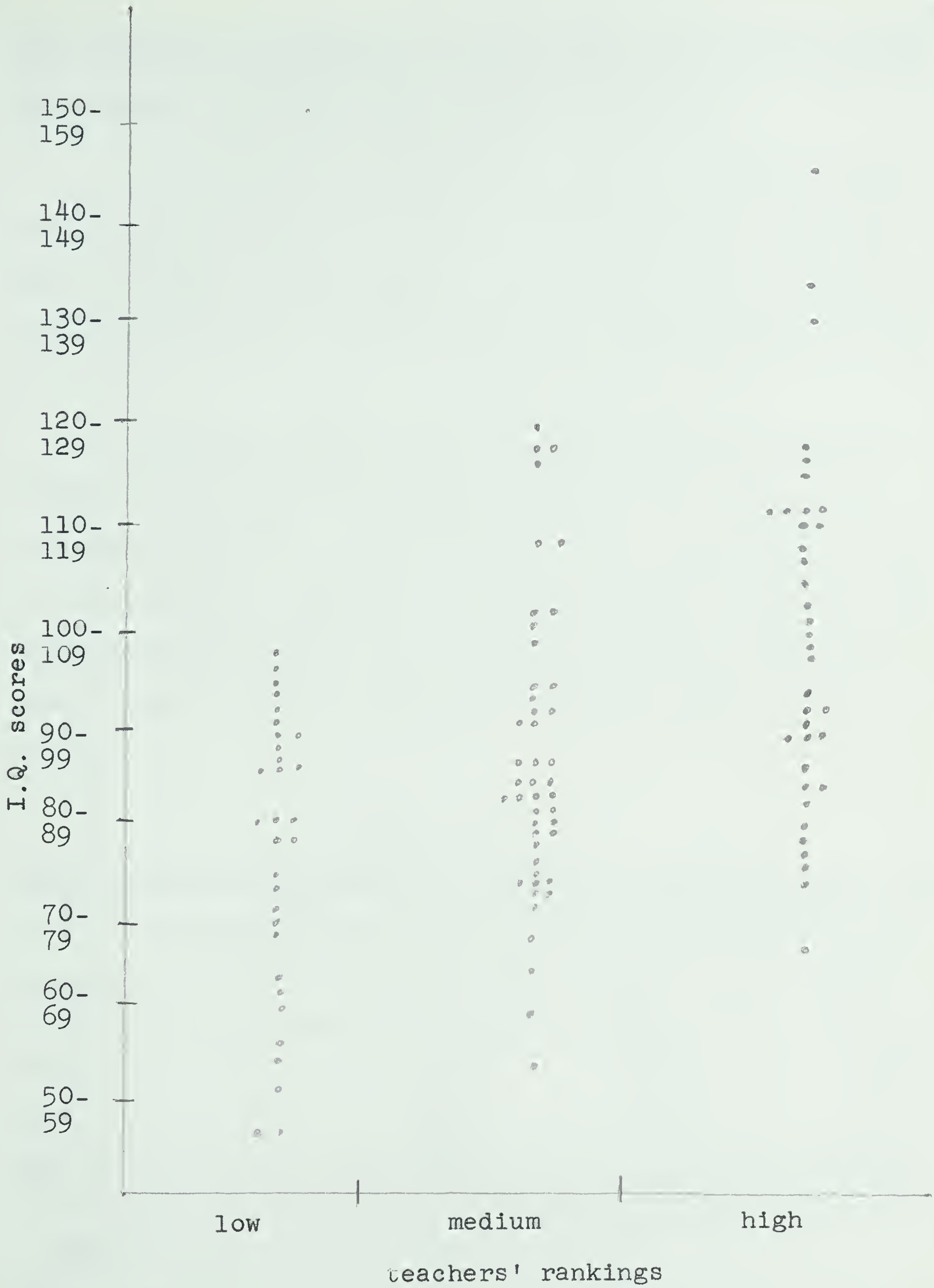

FIGURE 13

SCATTER DIAGRAM OF THE RELATIONSHIP BETWEEN TEACHERS RANKINGS AND INTELLIGENCE QUOTIENTS
(n=113)

THE RELATION OF MATHEMATICAL ACHIEVEMENT TO VARIABLES OTHER THAN LOGICAL PROCESSES AND TEACHERS' RANKINGS

The correlation coefficients of the relationship between mathematical achievement and intelligence, age, and sex are shown in TABLE VIII. The S.T.A.T. scores do correlate positively and significantly with intelligence and sex.

A further analysis of this relationship as seen in FIGURE 14 shows that children in this sample with an Intelligence Quotient of one hundred ten or better are likely to earn a total achievement score of a least fifty-one. Of the twenty-six pupils whose Intelligence Quotient was one hundred ten or better only three received scores below fifty-one.

TABLE VIII

CORRELATIONS OF MATHEMATICAL ACHIEVEMENT WITH I.Q.,AGE, AND SEX

S.T.A.T.	I.Q.	Age	Sex
total	.602**	.189	.327**
Part 1	.633**	.118	.215*
Part 2	.480**	.203*	.319**

* significant at the .05 level ($r \geqq .195$)

**significant at the .01 level ($r \geqq .254$)

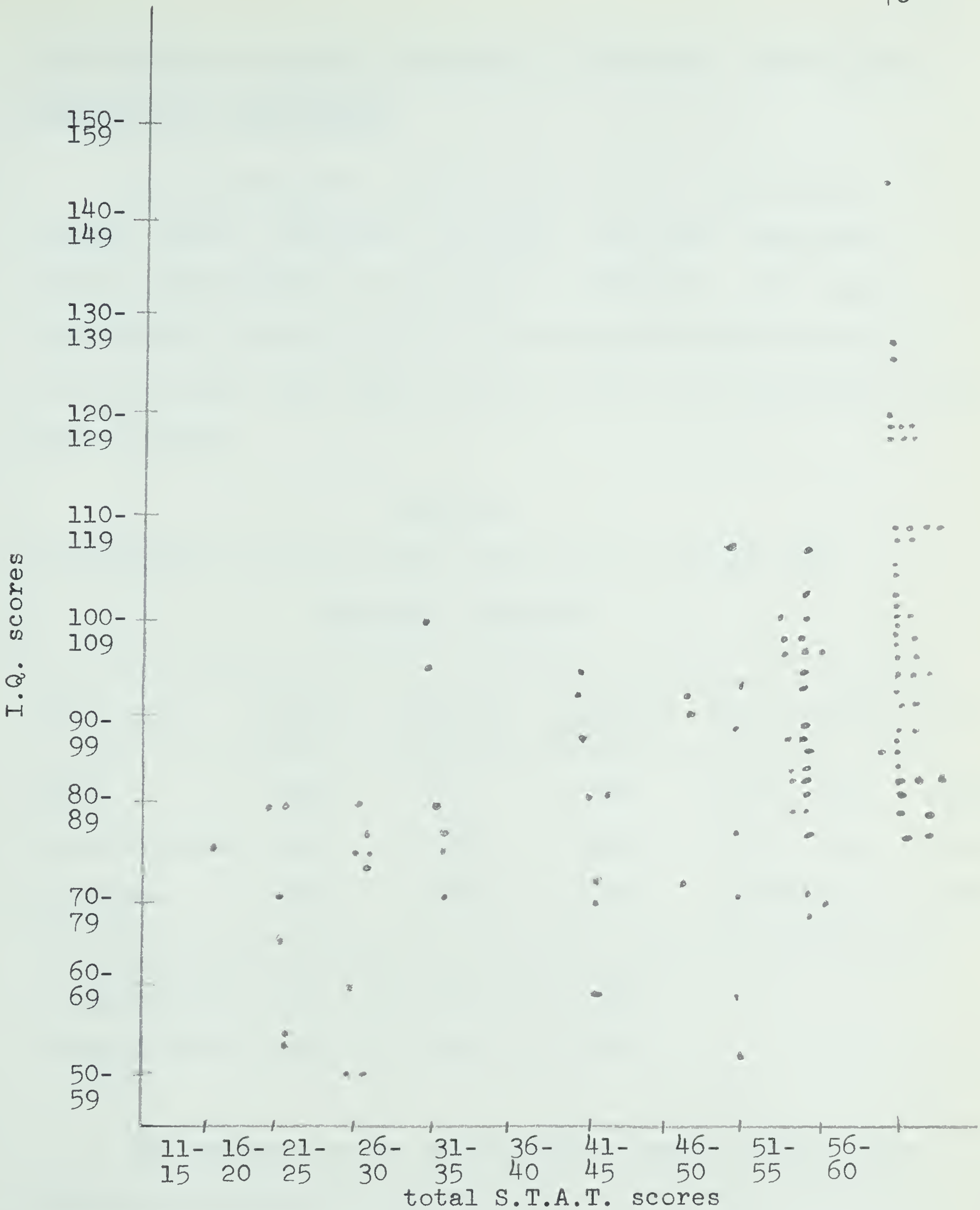

FIGURE 14

SCATTER DIAGRAM OF THE RELATIONSHIP BETWEEN S.T.A.T. AND INTELLIGENCE QUOTIENTS

(n=113)

THE RELATION OF LOGICAL PROCESSES TO TEACHERS' RANKINGS AND MATHEMATICAL ACHIEVEMENT

The correlation coefficients of the relationship between logical processes, teachers' rankings, and mathematical achievement are presented in TABLE IX. All logic test scores correlate positively and significantly with Part 1 S.T.A.T. scores and total S.T.A.T. scores, but not with S.T.A.T. Part 2 scores.

TABLE IX

CORRELATIONS OF LOGIC SCORES WITH S.T.A.T. SCORES AND TEACHERS' RANKINGS

Logic score	S.T.A.T. Part 1	S.T.A.T. Part 2	S.T.A.T. Total	High	Medium	Low
total	.289**	.111	.202*	.144	.136	-.302**
classification	.257**	.286**	.314**	.137	.147	-.305**
seriation	.470**	.265**	.377**	.347**	.060	-.428**

* significant at the .05 level ($r \geqq .195$)

**significant at the .01 level ($r \geqq .254$)

An analysis of the relationship between logical processes and mathematical achievement can be seen in the following scatter diagrams. FIGURE 15 pictures the relationship between total S.T.A.T. scores and total logic scores. The diagram shows that high logic scores are likely to indicate high achievement scores for the children in this sample. Of

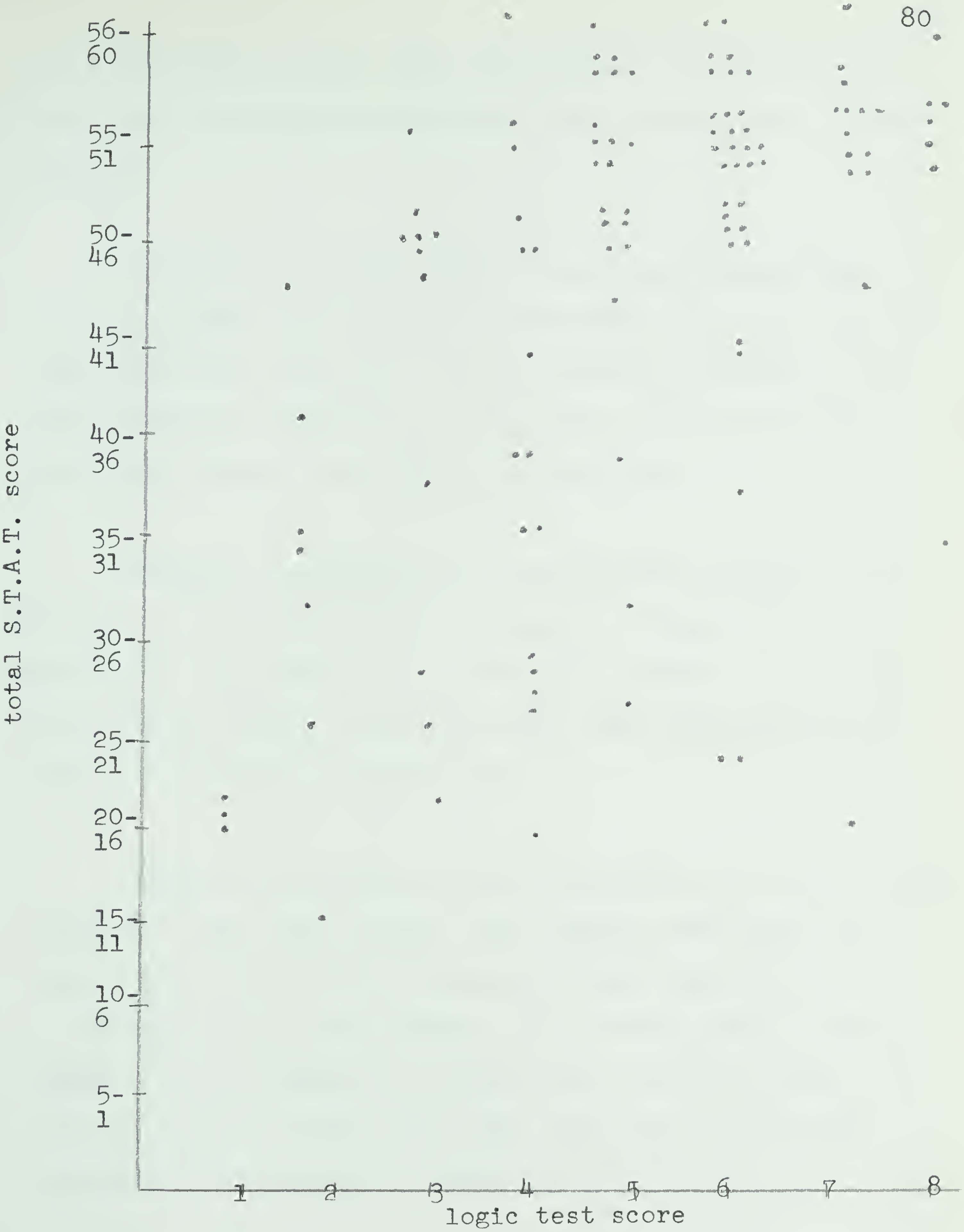

FIGURE 15

SCATTER DIAGRAM OF THE RELATIONSHIP BETWEEN LOGICAL PROCESSES AND MATHEMATICAL ACHIEVEMENT

(r=.202)

the twenty-three people who scored seven or better on the logic test, only three received a score below fifty on the S.T.A.T.

FIGURE 16 pictures the relationship between total S.T.A.T. scores and classification scores. Of the seventy-four people who received a score of three or better on the classification subsections of the logic test, only twenty received a score below fifty on the S.T.A.T.

FIGURE 17 represents the relationship between total S.T.A.T. scores and seriation scores. Of the forty-nine people who received a score of three or better on the seriation subsections of the logic test, only seven received a score below fifty on the S.T.A.T.

FIGURE 18 represents the relationship of S.T.A.T. Part 1 scores to logic test scores. The diagram shows that high logic scores are likely to indicate high scores on Part 1 of the mathematical achievement test for children in this sample. Of the twenty-two people who received a score of seven or better on the logic test, only five scored below twenty on Part 1 of the S.T.A.T.

FIGURE 19 pictures the relation between classificatory logic and scores on Part 1 of the S.T.A.T. Of the seventy-four pupils who scored three or better on the classification subsections of the logic test, thirty-seven of them

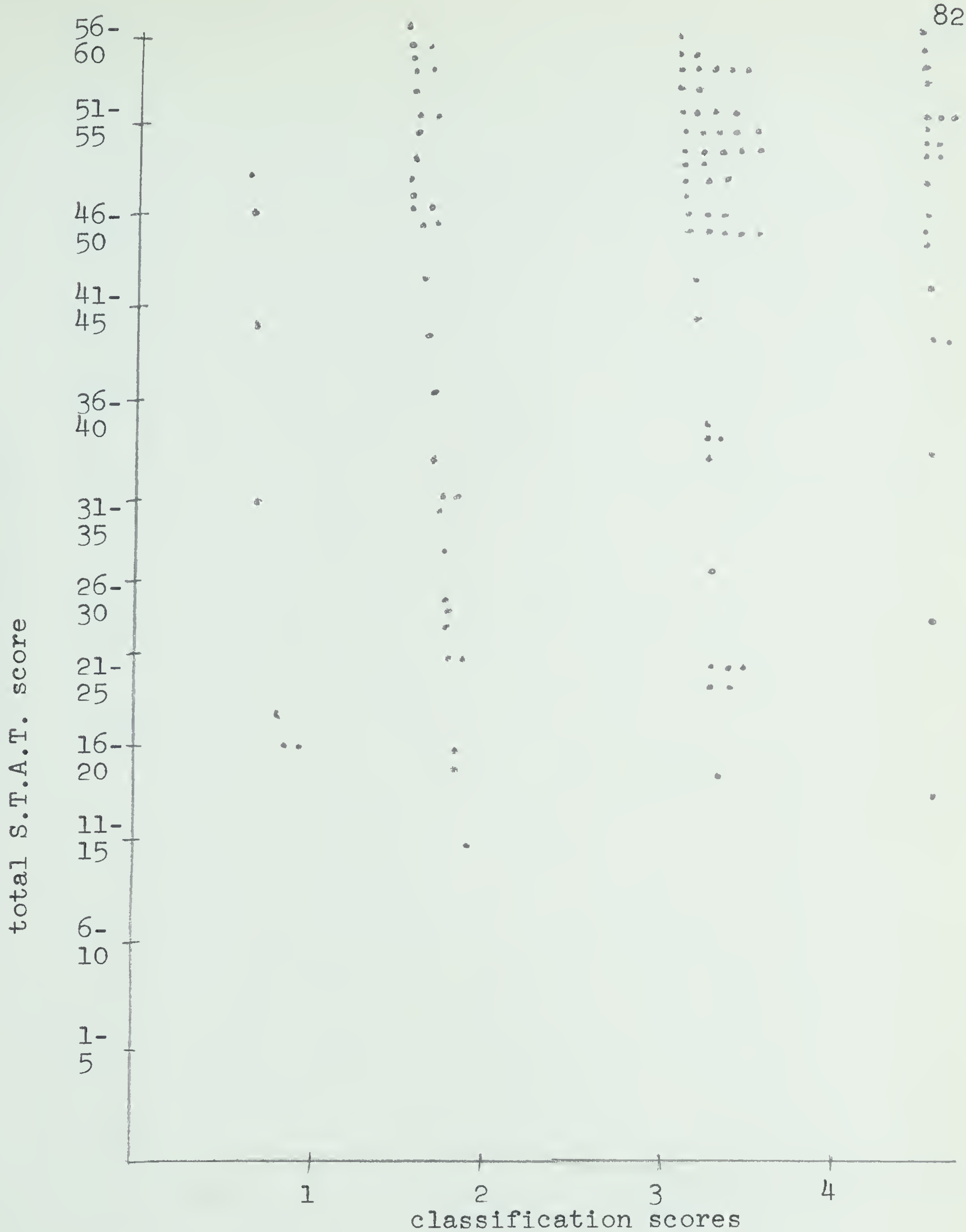

FIGURE 16

SCATTER DIAGRAM OF THE RELATIONSHIP BETWEEN CLASSIFICATORY LOGIC AND MATHEMATICAL ACHIEVEMENT

(r=.314)

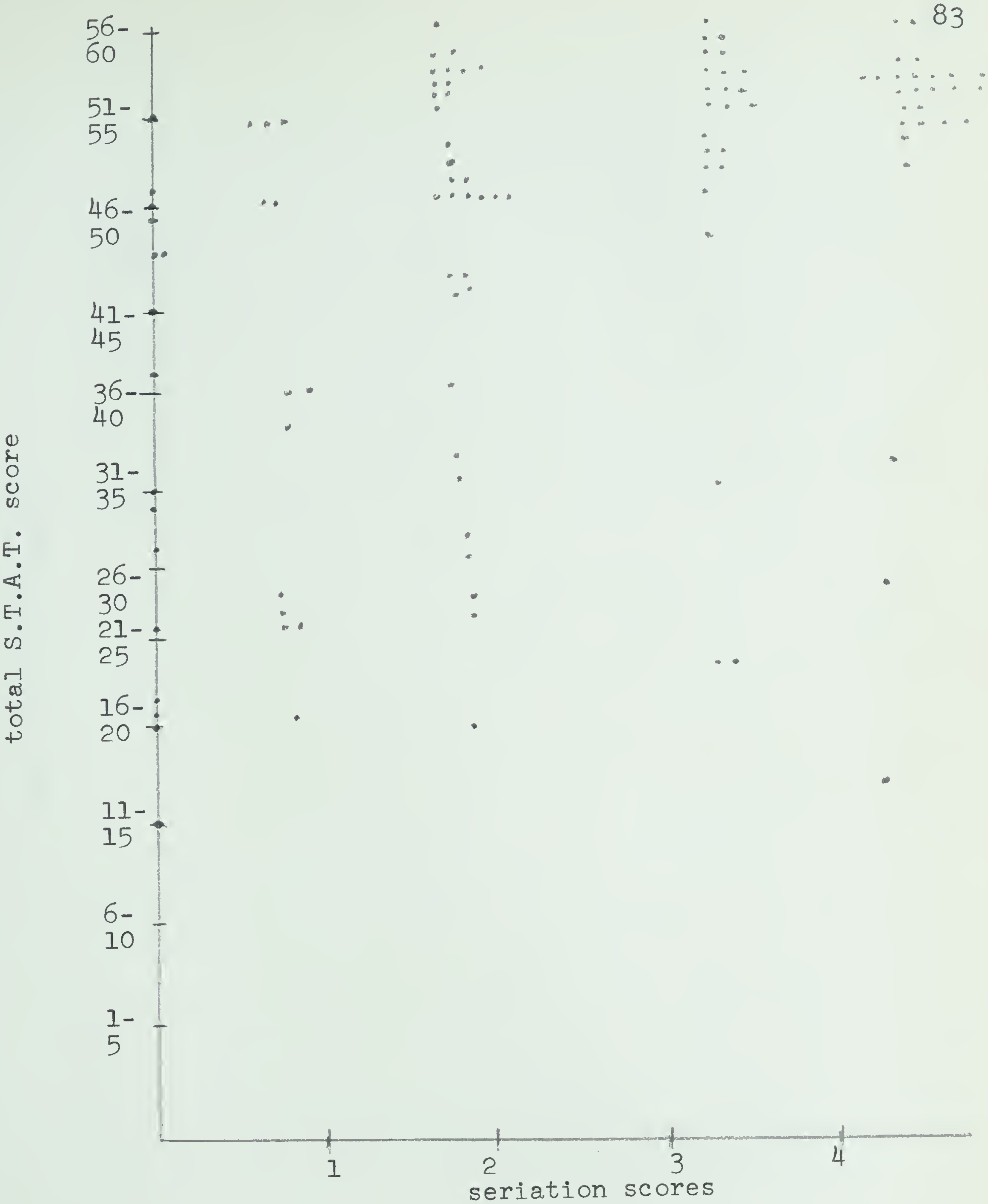

FIGURE 17

SCATTER DIAGRAM OF THE RELATIONSHIP BETWEEN SERIATION SCORES AND MATHEMATICAL ACHIEVEMENT

($r=.377$)

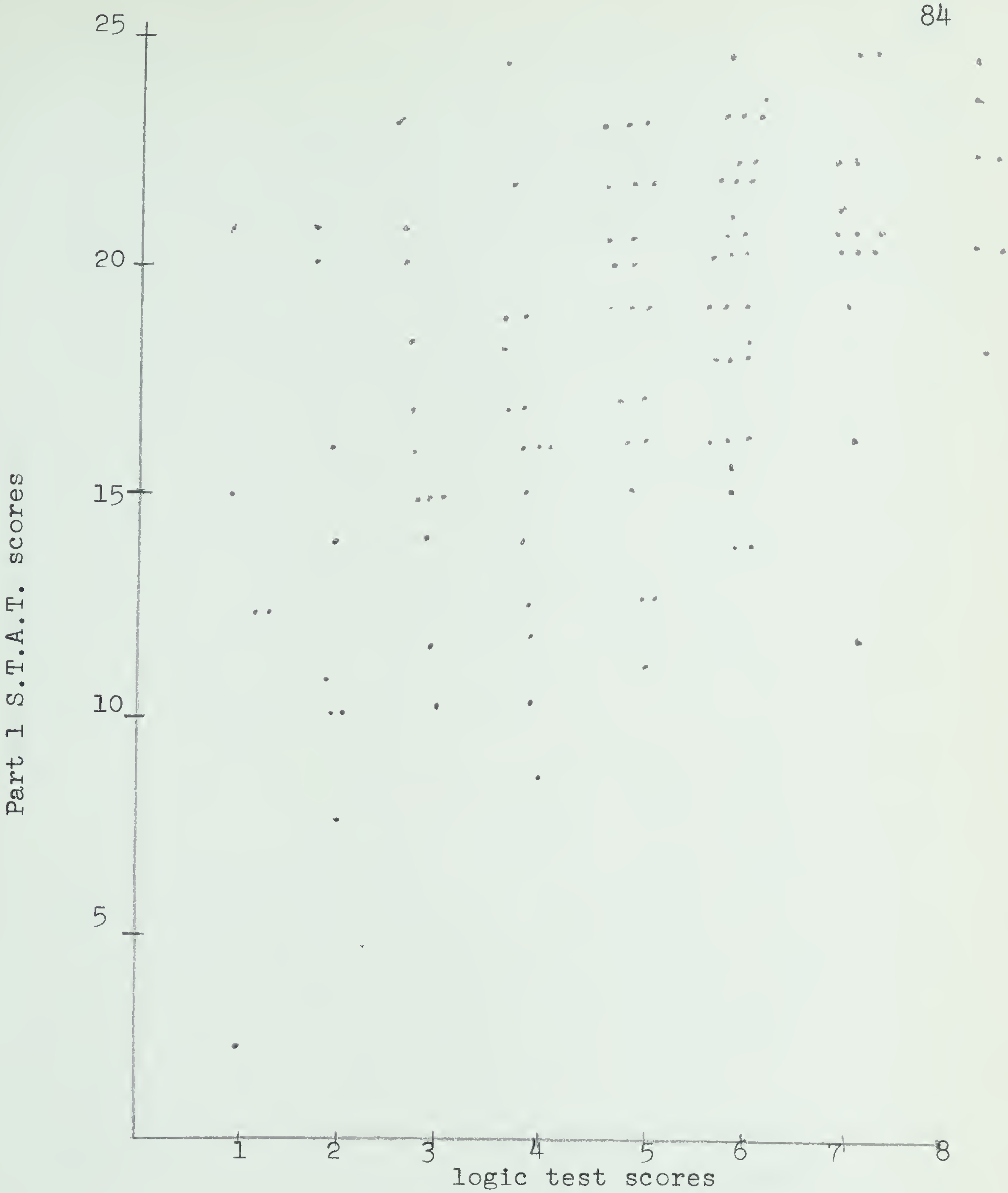

FIGURE 18

SCATTER DIAGRAM OF THE RELATIONSHIP BETWEEN LOGICAL PROCESSES

AND PART 1 S.T.A.T. SCORES

(r-.289)

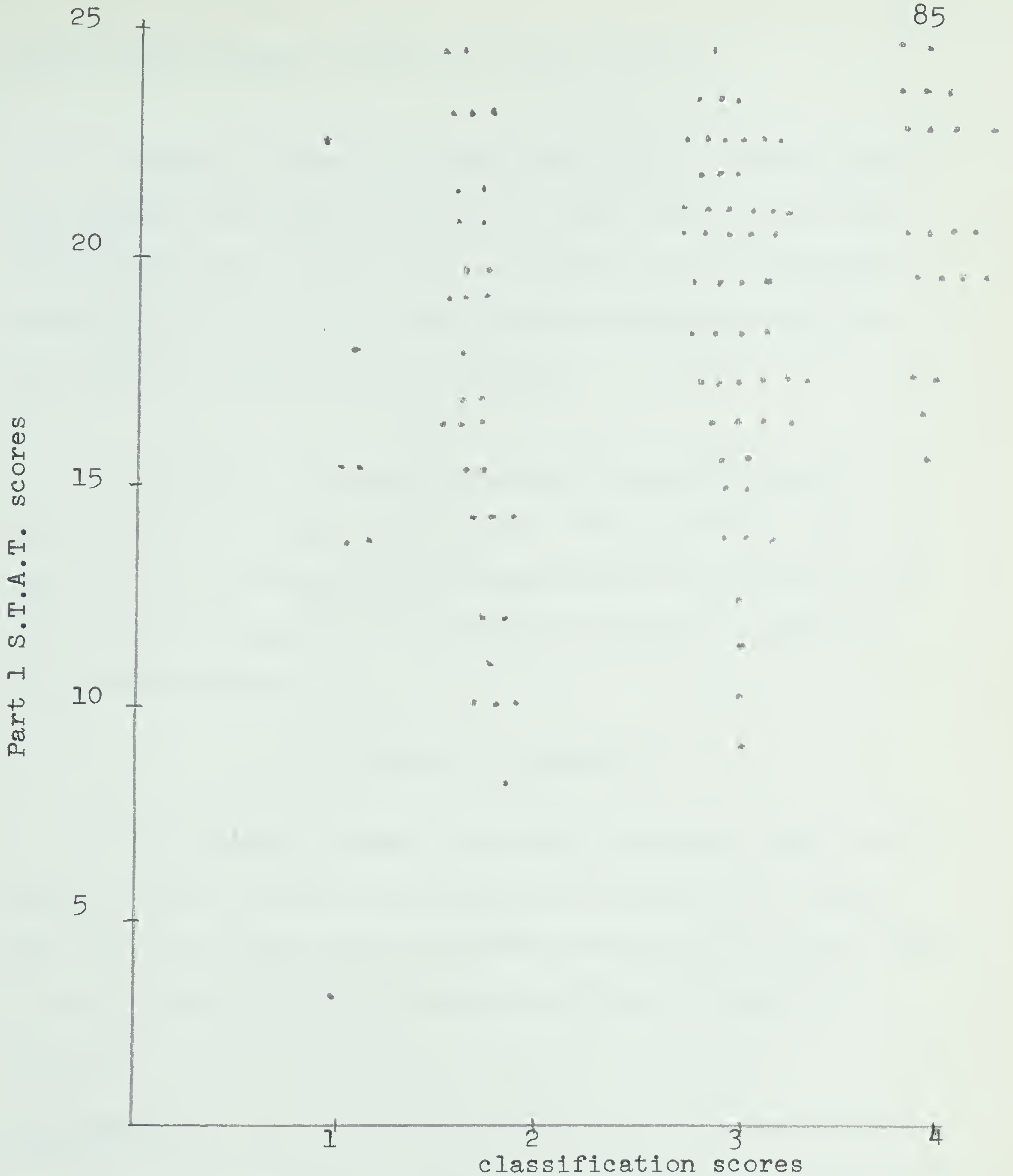

FIGURE 19

SCATTER DIAGRAM OF THE RELATIONSHIP BETWEEN CLASSIFICATORY LOGIC AND PART 1 S.T.A.T. SCORES

(r=.257)

scored below twenty on Part 1 of the S.T.A.T.

FIGURE 20 represents the relationship between seriation scores and scores on Part 1 of the S.T.A.T. Of the fifty pupils who scored three or better on the seriation subsections of the logic test fourteen of them scored below twenty on Part 1 of the S.T.A.T.

No scatter diagrams are shown relating S.T.A.T. Part 2 scores to logic test scores. The nature of this section of the mathematical achievement test is not seen to be an appropriate measure of arithmetic ability in terms of conceptualization.

V. SUMMARY OF RESULTS

The multiple linear regression technique requires the establishment of full and restricted models. To compare the predictive efficiency of these models an F test was used. A brief summary of the finding regarding the hypothesis follows.

HYPOTHESIS 1

The ability to classify and seriate is not a significant predictor of mathematical achievement, when differences in ability are taken into account.

This hypothesis was accepted. The F value for the corresponding models was found to be 1.16 and not significant.

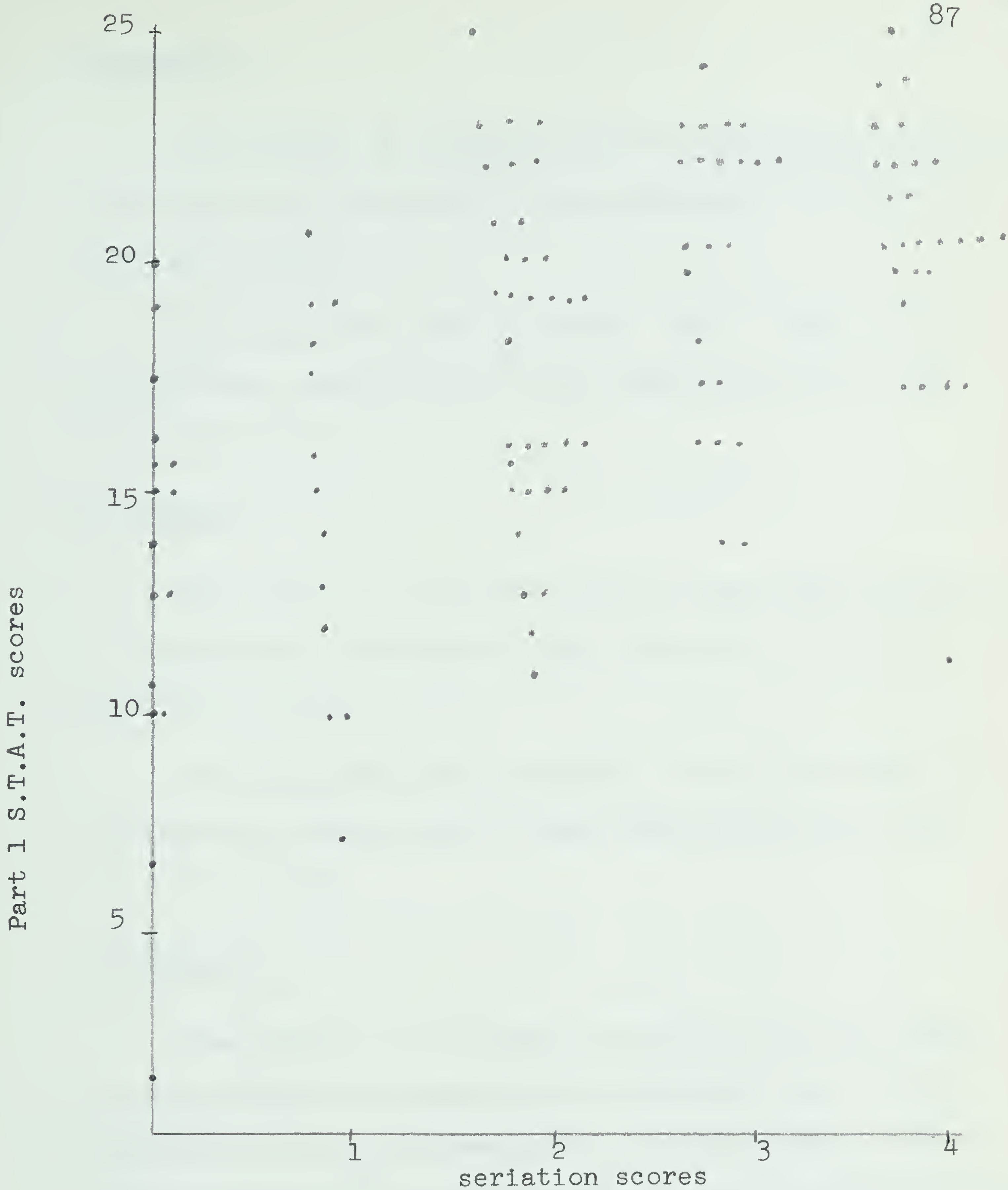

FIGURE 20

SCATTER DIAGRAM OF THE RELATIONSHIP BETWEEN SERIATION AND PART 1 S.T.A.T. SCORES

$(r=.470)$

HYPOTHESIS 2

The ability to classify is not a significant predictor of mathematical achievement, when differences in ability are taken into account.

This hypothesis was rejected. The F value for the corresponding models was 5.53 and significant at the .05 level ($F \geqq 3.94$).

HYPOTHESIS 3

The ability to seriate is not a significant predictor of mathematical achievement, when differences in ability are taken into account.

This hypothesis was rejected. The F value for the corresponding models was 6.49 and significant at the .05 level ($F \geqq 3.94$).

HYPOTHESIS 4

The ability to classify and seriate is not a significant predictor of mathematical achievement test scores, taking into account differences in ability, when the knowledge of and ability to apply certain basic concepts of arithmetic is being examined.

This hypothesis was rejected. The F value for the corresponding models was 5.16 and significant at the .05 level ($F \geqq 3.94$).

HYPOTHESIS 5

The ability to classify is not a significant predictor of mathematical achievement test scores, taking into account differences in ability, when the knowledge of and ability to apply certain basic concepts of arithmetic is being examined.

This hypothesis was accepted. The F value for the corresponding models was found to be 2.18 and not significant.

HYPOTHESIS 6

The ability to seriate is not a significant predictor of mathematical achievement test scores, taking into account differences in ability, when the knowledge of and ability to apply certain basic concepts of arithmetic is being examined.

This hypothesis was rejected. The F value for the corresponding models was 16.09 and significant at the .01 level ($F \geq 6.90$).

HYPOTHESIS 7

The ability to classify and seriate is not a significant predictor of mathematical achievement test scores, taking into account differences in ability, when the knowledge of basic number facts is being examined.

This hypothesis was accepted. The F value for the corresponding models was found to be .028 and not significant.

HYPOTHESIS 8

The ability to classify is not a significant predictor of mathematical achievement test scores, taking into account differences in ability, when the knowledge of basic number facts is being examined.

This hypothesis was rejected. The F value for the corresponding models was 4.60 and significant at the .05 level ($F \geq 3.94$).

HYPOTHESIS 9

The ability to seriate is not a significant predictor of mathematical achievement test scores, taking into account differences in ability, when the knowledge of basic number facts is being examined.

This hypothesis was accepted. The F value for the corresponding models was found to be 1.85 and not significant.

HYPOTHESIS 10

The ability to seriate is not a significant predictor of success on test items dealing with seriation.

This hypothesis was rejected. The F value for the corresponding models was 12.49 and significant at the .01 level ($F \geq 6.90$).

HYPOTHESIS 11

The ability to classify is not a significant predictor of success on test items dealing with seriation.

This hypothesis was accepted. The F value for the corresponding models was found to be 3.74 and not significant.

HYPOTHESIS 12

The ability to seriate is not a significant predictor of success on test items dealing with classification.

This hypothesis was rejected. The F value for the corresponding models was 16.72 and significant at the .01 level ($F \geq 6.90$).

HYPOTHESIS 13

The ability to classify is not a significant predictor of success on test items dealing with classification.

This hypothesis was rejected. The F value for the corresponding models was 6.73 and significant at the .05 level ($F \geq 3.94$).

HYPOTHESIS 14

The ability to classify and seriate is not a significant predictor of success on test items dealing with seriation.

This hypothesis was rejected. The F value for the

corresponding models was 6.40 and significant at the .05 level ($F \geq 3.94$).

HYPOTHESIS 15

The ability to classify and seriate is not a significant predictor of success on test items dealing with classification.

This hypothesis was rejected. The F value for the corresponding models was 4.61 and significant at the .05 level ($F \geq 3.94$).

HYPOTHESIS 16

Teachers' subjective rankings of their students' mathematical ability is not a significant predictor of the students' ability to classify and seriate, when differences in ability are taken into account.

This hypothesis was rejected. The F value for the corresponding models was 3.47 and significant at the .05 level ($F \geq 3.09$).

HYPOTHESIS 17

Teachers' subjective rankings of their students' mathematical ability is not a significant predictor of the students' ability to classify, when differences in ability are taken into account.

This hypothesis was rejected. The F value for the corresponding models was 3.17 and significant at the .05 level ($F \geqq 3.09$).

HYPOTHESIS 18

Teachers' subjective rankings of their students' mathematical ability is not a significant predictor of the students' ability to seriate, when differences in ability are taken into account.

This hypothesis was rejected. The F value for the corresponding models was 8.44 and significant at the .01 level ($F \geqq 4.82$).

SUMMARY OF RESULTS

Significant, positive correlations were found between logic test scores and intelligence, S.T.A.T. scores and sex and intelligence, and logic test scores and scores on the total S.T.A.T. and Part 1 of the S.T.A.T.

Significant, negative correlations were found between logic test scores and teachers' rankings of students with low mathematical ability.

Few significant relationships were found between the variables of logic test scores, teachers' rankings, age, and sex. S.T.A.T. scores also showed no correlation with age.

CHAPTER V

SUMMARY, CONCLUSIONS, IMPLICATIONS, AND SUGGESTIONS FOR FURTHER RESEARCH

It was the purpose of this study to explore the possibility that mastery of classificatory and serial logic enables children to achieve better in mathematics.

The sample consisted of one hundred thirteen children from five first grade classrooms in the Edmonton Public School System. These classrooms were randomly selected. The age range of the sample was from seventy-nine to eighty-eight months. Intelligence Quotients varied from fifty-seven to one hundred fifty-three as measured by the Detroit Beginners First Grade Intelligence Test. Fathers' occupations, as indicated on the schools' records, were rated according to the "Blishen Occupational Class Scale." Almost half of the children in the sample came from middle class homes. The remainder of the children were almost evenly distributed in the lower and upper economic classes.

Teachers' rankings were collected of each student's mathematical ability. During the final week of May, 1968 a test of logic which was adapted from the procedures used by Piaget (1964) was individually administered to each child. This test was designed to assess the child's ability to classify, form hierarchical class structures, seriate along two dimensions, and order elements. One week later classroom teachers

administered the Seeing Through Arithmetic Test (S.T.A.T.).

The data analysis was facilitated by the Division if Educational Research Services, Faculty of Education, at the University of Alberta.

The remainder of this chapter contains the conclusions reached, the educational implications of these conclusions, and some suggestions for further research.

I. CONCLUSIONS

After testing the hypothesis the following conclusions were formed.

The ability to coordinate the use of classification and seriation is not a statistically significant predictor of mathematical achievement as reflected by the total S.T.A.T. score. This finding may have been influenced by the nature of the particular achievement test. Additional analysis revealed that the ability to coordinate classification and seriation was a significant predictor of the scores on Part 1 of the S.T.A.T. but not on Part 2. Part 2 of the S.T.A.T. tested knowledge of basic number facts. Almost half of the children in the sample scored thirty-four or above, out of a possible thirty-six, on Part 2. This may indicate that this test section is a poor measure of the ability to conceptualize and apply mathematical principles. No further conclusions will be made concerning Part 2 of the S.T.A.T. for that reason.

In any case, the fact that the coordination of logical processes was not a predictor of Part 2 scores may have biased the general findings. There were more test items on Part 2 than on Part 1. Therefore, when the scores of both parts were combined the significant relationship involving Part 1 scores may have been negated by the insignificant relationship involving Part 2 scores. This would result in the total score being insignificantly related to the child's ability to coordinate the processes of classification and seriation.

An alternative explanation to the finding that the coordination of logical processes is not a significant predictor of mathematical achievement is provided by the statistical analysis used. When classificatory logic alone was examined, it was found to be a significant predictor of mathematical achievement. Seriation ability was also found to be a significant predictor of mathematical achievement. The number of people in the group who could classify and the number of people in the group who could seriate was close to the fifty per cent mark in each case. More than one-half of the children could use classificatory logic and almost one-half could seriate. Therefore, groupings were large enough to allow significant relationships to show up if they did exist. When children who could coordinate the processes of classification and seriation were placed in one group a more stringent criteria than might have been appropriate was applied. The membership of this group was small. Only

one-fifth of the children in this sample were credited with being able to coordinate logical processes. If a significant relationship did exist it had no chance to show up because of the small number of persons involved.

Concentrating now on the prediction of Part 1 S.T.A.T. scores, it was found that classificatory logic was not a statistically significant predictor of these scores while seriation was. However, an instance was not found in which a child could classify yet received a low score on Part 1 of the S.T.A.T.

An attempt was made to assess the relative independence of classificatory logic and sequential ordering ability. It was found that classificatory logic was a significant predictor of success on classificatory S.T.A.T. items ($r=.239$) but not on S.T.A.T. items which were seriation oriented ($r=.181$). Therefore, the ability to classify seems to be independent of the ability to seriate. It was also discovered that seriation was a significant predictor of success on both types of S.T.A.T. items (classification items - $r=.362$, seriation items - $r=.318$). Seriation seems to be somewhat dependent upon classificatory logic. This conclusion becomes more meaningful when one considers that before elements can be ordered there must exist some basis for comparison. Two abilities are needed before ordering can occur. One is an ability to group all elements into one category. The other is an ability to form smaller categories based on

the differences between the elements. Therefore, for some children in the sample the ability to seriate necessitated a prior mastery of classification.

As a result of the investigation it was also concluded that teachers' subjective evaluations of students' arithmetic ability are statistically significant predictors of the ability of those students to classify, seriate, and coordinate the use of both these logical processes. However, this conclusion must be evaluated in light of the very tenuous nature of teachers' rankings.

Finally, a conclusion was reached concerning preference for classificatory criterion. Four boys and one girl chose color as the only classificatory base. Eighty-six of the children chose form. This finding is in accordance with that of Kagen and Lemkin (1961) who discovered that form is preferred over color and size as a basis for similarity grouping by both boys and girls.

II. IMPLICATIONS

Certain implications for the mathematics instruction of first grade children may be drawn.

The ability to classify, seriate, and coordinate these processes is connected with mathematical achievement. This correlation exists not only with arithmetic achievement test scores, but also with teachers' rankings of pupils' arithmetic

ability. Relations which involved the results of Part 2 of the S.T.A.T. were found not to be significant. The implication here is that testing for knowledge of number facts does not assess how efficiently certain cognitive processes are used. It would be unwise for a teacher to assume on the basis of an achievement test score that a child understands the mathematical concepts to which he had been introduced in the first grade. Tests patterned after Part 1 of the S.T.A.T. may serve as better indicators of the operational levels of children.

The findings suggest than an occasional spot check of children's reasoning patterns be conducted by the teacher. This check could employ the four tasks used in this study. The materials needed are simply constructed, and the time factor is minimal. Hopefully, the assessment made for each pupil would indicate whether he was ready for beginning number work. If a child has a limited basis for classifying, it is doubtful that he will meaningfully comprehend number properties such as "twoness" or "threeness." A child who cannot order concrete objects will be unsuccessful when trying to order numbers on the basis of the abstract property of being "one greater than." It is advisable to delay beginning number work until children are able to efficiently use the logical processes of classification and seriation.

Teachers' evaluations of students' arithmetic ability reflect children's abilities to classify and seriate. This

may mean that teachers are intuitively attempting to assess children's arithmetic ability in terms of cognition. What is needed is a more objective, reliable, valid instrument to do the same job.

Classificatory logic was found not to be useful in answering test items based on sequential ordering operations. In contrast to this it was discovered that the ability to seriate does help children to answer questions which seem to require only classificatory logic. This may mean that classificatory logic is composed of some thought patterns which provide a necessary, but not sufficient basis for sequential ordering. It is reasonable to suggest that before a child can order elements he must be able to see some common feature among them by which they differ. This notion is reflected in the Seeing Through Arithmetic program which introduces the child to cardinal number before ordinal number.

III. SUGGESTIONS FOR FURTHER RESEARCH

It was found that the ability to coordinate the processes of classification and seriation was not correlated with mathematical achievement test scores. This may have occurred because of the nature of the mathematical achievement test used or because of the method of statistical analysis. Further investigation is needed to assess the predictive efficiency of classification and seriation in light of a more valid measure of mathematical achievement. Additional data analysis should also be carried out in the future.

This would more clearly define the interaction between the processes of classification and seriation. The assumption that there is indeed something extra which differentiates between being able to classify and seriate and being able to deal with both these processes simultaneously needs further consideration.

Additional work is necessary in the field of evaluation. Examinations are needed which will go beyond the surface of rote learning and assess the concepts which have been assimilated. Tests of this type would also be more efficient diagnostic instruments, because they would indicate which area of the child's mathematical knowledge is missing. This would lessen the chance that later mathematical experiences were being built on a shakey foundation.

The most important need arising from this research is for the establishment of learning experiences designed specifically to help children develop the needed thought processes. Future research must concern itself with whether the process of logical thinking can be hastened or even aided by certain experiences. Perhaps these phenomena are strictly maturational and will not be enhanced by concrete experiences or training. If this is found not to be the situation, then materials and methods must be devised to help the child develop the needed operations before beginning formal number work.

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APPENDIX

APPENDIX

A DETAILED DESCRIPTION OF THE INDIVIDUAL TESTING PROCEDURE

When the child entered the room he was asked his name and how he was feeling that morning. He was then guided to a table and placed conviently at the center of one of its sides.

Then the child was told to "put together in piles all the things that are the same." If there was any question by the child as to how this was to be done, he was told to "do it any way you like." While the child was engaged in Task I the necessary data were extracted from his cumulative record card. At the same time the child's behavior was being observed. This was done to avoid any pressure upon the child to perform in a certain manner. After the child was finished he was asked, "Can you put them together another way? Is there anything else about some of these things that is the same?"

After Task I was completed the child was asked to move further down the table to the indicated place. The child was then shown a circle and a square and was asked to name them. The names he gave the shapes were the names used in the questioning that followed. The child was asked to look carefully at the array before him. He was then asked if all the squares were orange and if all the blue ones were squares.

The child's attention was next directed to a display

of pictures. All the pictures running horizontally were designated as rows. This was told to the child a number of times, and he was asked to show where a row was. The child was then asked to "find the second picture in the second row." After doing this he was asked to locate the "fifth picture in the third row." He was then given some positive, verbal reinforcement.

The child was finally asked to move to the end of the table. He was instructed to "put these cans in a line, so that the tallest can is at one end and the shortest can is at the other end of the line." A hand gesture was then made to trace a descending progression in height along a linear dimension. After the child had completed this activity he was asked to reconsider what he had done. He was asked if the tallest can was at one end and the shortest can at the other end. If a proper sequence was established, five additional cans were given to the child, and he was told to "put these cans in that line where you think they belong." The child was thanked and instructed to return to his classroom.